IEEE Guide for Design of Substation Rigid-Bus Structures

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Abstract: Rigid-bus structures for outdoor and indoor, air-insulated, and alternating-current substations are covered. Portions of this guide are also applicable to strain-bus structures or direct-current substations, or both. Ampacity, radio influence, vibration, and forces due to gravity, wind, fault current, and thermal expansion are considered. Design criteria for conductor and insulator strength calculations are included.
Keywords: ampacity, bus support, mounting structure, rigid-bus structures, strain-bus structure

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Introduction

(This introduction is not part of IEEE Std 605-1998, IEEE Guide for Design of Substation Rigid-Bus Structures.)

Substation rigid-bus structures are being applied by electrical utilities and other industries. Such structures usually reduce substation heights and have greater ampacity and lower corona than single-conductor strain-bus structures.

This guide presents an integrated design approach with methods for calculating the forces to which rigid-bus structures are subjected.

The data in this guide are taken from empirical and theoretical sources that can form the basis of a good design by a knowledgeable design engineer. The guide is not intended as a rigid procedural design guide for the inexperienced. Some of the empirical methods presented in this guide are based on experience.

Future work is required to give the design engineer a better understanding of certain portions of the design process, such as the flexibility of support structures and insulator overload factors.

The Working Group D3 responsible for the preparation of this guide had the following membership at the time this guide was submitted for approval:

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IEEE Guide for Design of Substation Rigid-Bus Structures

1. Overview

1.1 Scope

The information in this guide is applicable to rigid-bus structures for outdoor and indoor, air-insulated, and alternating-current substations. Portions of this guide are also applicable to strain-bus structures or direct-current substations, or both. Ampacity, radio influence, vibration, and forces due to gravity, wind, fault current, and thermal expansion are considered. Design criteria for conductor and insulator strength calculations are included. This guide does not consider

a) The electrical criteria for the selection of insulators
b) The seismic forces to which the substation may be subjected
c) The design of mounting structures

1.2 Purpose

Substation rigid-bus structure design involves electrical, mechanical, and structural considerations. It is the purpose of this guide to integrate these considerations into one document.

Special consideration is given to fault current-force calculations. Factors considered include the decrement of the fault current, the flexibility of supports, and the natural frequency of the bus. These factors are mentioned in ANSI C37.32-1996, but are not taken into consideration in the equations presented in that standard.

2. References

This guide shall be used in conjunction with the following publications. If the following publications are superseded by an approved revision, the revision shall apply.

ANSI C29.1-1988 (R1996), American National Standard Test Methods for Electrical Power Insulators.[1]

[1]ANSI publications are available from the Sales Department, American National Standards Institute, 11 West 42nd Street, 13th Floor, New York, NY 10036, USA.

ANSI C29.9-1983 (R1996), American National Standard for Wet-Process Porcelain Insulators (Apparatus, Post Type).

ANSI C37.32-1996, American National Standard for High-Voltage Air Disconnect Switches Interrupter Switches, Fault Initiating Switches, Grounding Switches, Bus supports and Accessories Control Voltage Ranges—Schedule of Preferred Ratings, Construction Guidelines and Specifications.

ASCE 7-95, Minimum Design Loads for Buildings and Other Structures.[2]

ASTM B188-96, Standard Specification for Seamless Copper Bus Pipe and Tube.[3]

ASTM B241/B241M-96, Standard Specification for Aluminum and Aluminum-Alloy Seamless Pipe and Seamless Extruded Tube.

IEEE Std C2-1997, National Electrical Safety Code.[4]

IEEE Std C37.30-1997, IEEE Standard Requirements for High-Voltage Air Switches.

IEEE Std 100-1996, IEEE Standard Dictionary of Electrical and Electronics Terms.

IEEE Std 693-1997, IEEE Recommended Practice for Seismic Design of Substations.

NEMA CC 1-1993, Electric Power Connectors for Substations.[5]

NEMA 107-1988 (R1993), Methods of Measurement of Radio-Influence Voltage (RIV) of High-Voltage Apparatus.

NFPA 70-1996, National Electrical Code.[6]

3. Definitions

The following definitions apply specifically to the subject matter of this guide:

3.1 bus structure: An assembly of bus conductors, with associated connection joints and insulating supports.

3.2 bus support: An insulating support for a bus.

NOTE—A bus support includes one or more insulator units with fittings for fastening to the mounting structure and for receiving the bus.

3.3 mounting structure: A structure for mounting an insulating support.

3.4 rigid-bus structure: A bus structure comprised of rigid conductors supported by rigid insulators.

3.5 strain-bus structure: A bus structure comprised of flexible conductors supported by strain insulators.

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[2]ASCE publications are available from the American Society of Civil Engineers, 1801 Alexander Bell Drive, Reston, VA 20191-4400, USA.

[3]ASTM publications are available from the American Society for Testing and Materials, 100 Barr Harbor Drive, West Conshohocken, PA 19428-2959, USA.

[4]IEEE publications are available from the Institute of Electrical and Electronics Engineers, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331, USA.

[5]NEMA publications are available from the National Electrical Manufacturers Association, 1300 N. 17th St., Ste. 1847, Rosslyn, VA 22209, USA.

[6]NFPA publications are available from Publications Sales, National Fire Protection Association, 1 Batterymarch Park, P.O. Box 9101, Quincy, MA 02269-9101, USA.

4. The design problem

The design problem considered in this guide is the selection of rigid-bus structure components and their arrangements. For a safe, reliable, and economic design, the components and their arrangements should be optimized to satisfy the design conditions.

The design conditions will establish minimum electrical and structural performance. These conditions are dependent upon the characteristics of the power system involved and the location of the substation. The design conditions specify the following:

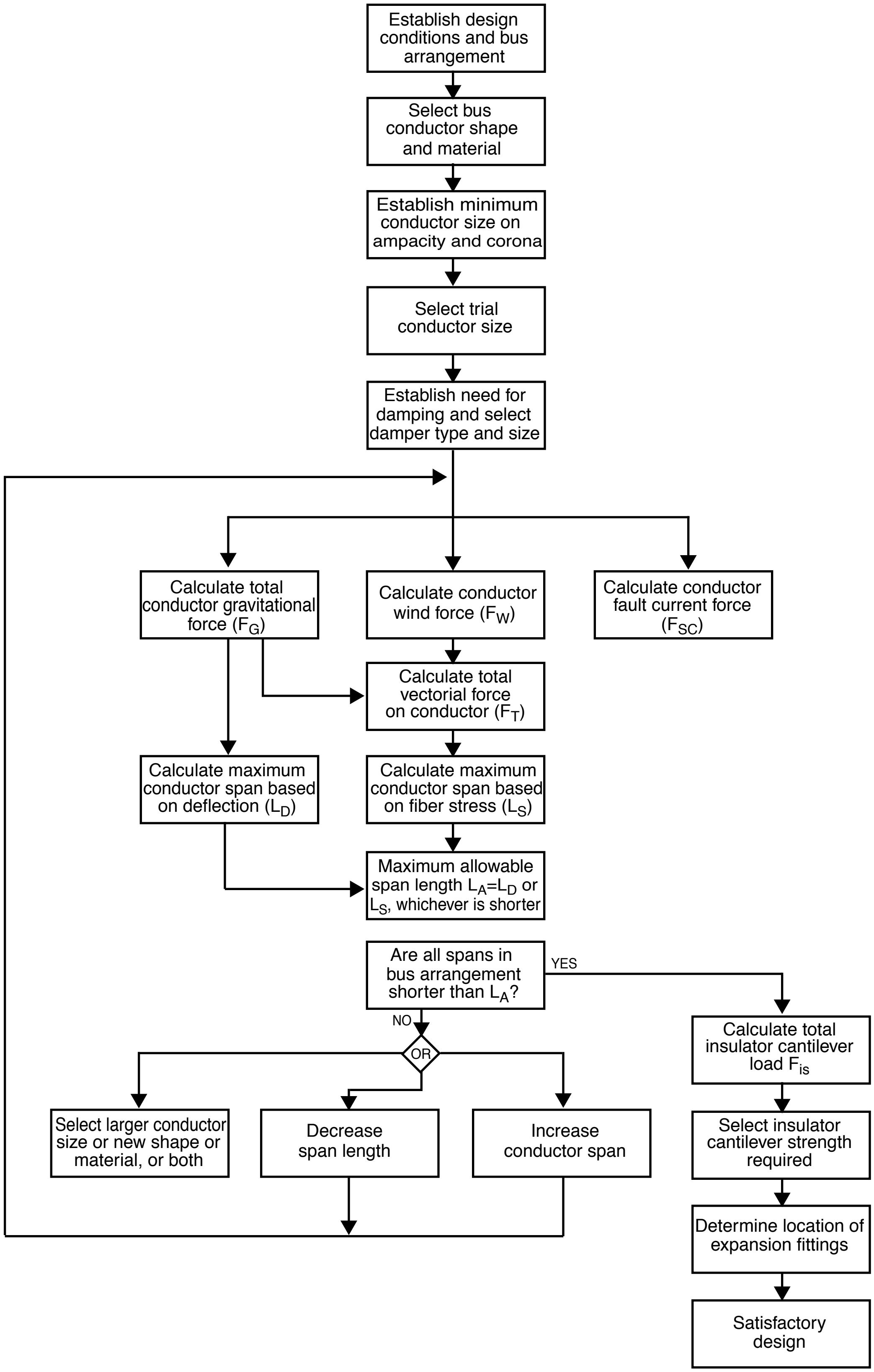
a) Ampacity requirements
b) Maximum anticipated fault current
c) Maximum operating voltage
d) Maximum anticipated wind speeds
e) Maximum expected icing conditions combined with wind
f) Altitude of the substation site
g) Basic substation layout

The selection of conditions acting simultaneously on the bus structure (that is, fault current, extreme wind, combined wind and ice, or a combination of these) involves probability, and some risk is involved in their selection. The design engineer should consider the risks to life, property, and system operation when the design conditions are selected.

Design conditions should also be specified for the electrical performance of insulators. If the substation is located in an area of possible seismic activity, additional design conditions should be established. IEEE Std 693-1997 and the seismic zone maps in ASCE 7-95 may be used to establish these seismic design conditions.

The actual design can begin after the design conditions are firmly established. Because of the various bus-structure components available to the designer and their various possible physical arrangement, the design becomes an iterative process. This iterative process is interrelated by conductor ampacity, suppression of radio influence, elimination of conductor vibrations, and structural integrity (see Figure 1).

It should be noted that a guide is presently being developed by the ASCE that will address the structural aspects of rigid-bus design. When approved, this ASCE guide may be used to verify the structural aspects of this guide.



NOTE—This diagram assumes that maximum span length is not limited by aeolian vibration.

Figure 1—Design process for horizontal rigid bus

5. Ampacity

The ampacity requirement of the bus conductor is usually determined by either the electrical system requirements or the ampacity of the connected equipment. Conductor ampacity is limited by the conductor's maximum operating temperature. Excessive conductor temperatures may anneal the conductor, thereby reducing its strength, or may damage connected equipment by the transfer of heat. Excessive temperatures may also cause rapid oxidation of the copper conductor.

5.1 Heat balance

The temperature of a conductor depends upon the balance of heat input and output. For balance, the heat input due to I^2R and solar radiation equals the heat output due to convection, radiation, and conduction. The heat balance may be expressed as

$$I^2RF + q_s = q_c + q_r + q_{cond} \tag{1}$$

Solving the current for a given conductor temperature rise is

$$I = \sqrt{\frac{q_c + q_r + q_{cond} - q_s}{RF}} \tag{2}$$

where

I = current for the allowable temperature rise, A
R = direct-current resistance at the operating temperature, Ω/m [Ω/ft]
F = skin-effect coefficient
q_s = solar heat gain, W/m [W/ft]
q_c = convective heat loss, W/m [W/ft*]
q_r = radiation heat loss, W/m [W/ft]
q_{cond} = conductive heat loss, W/m [W/ft*]

* Values for convective or conductive heat gains on the right side of Equation (2) are entered as negative numbers.

5.1.1 Effective resistance, RF

A conductor's effective resistance at a given temperature and frequency is the direct-current resistance *R* modified by the skin-effect coefficient *F*. These values may be obtained from published data.

5.1.2 Solar heat gain, q_s

The amount of solar heat gained is a function of

a) The total solar and sky radiation
b) The coefficient of solar absorption for the conductor's surface
c) The projected area of the conductor
d) The altitude of the conductor above sea level
e) The orientation of the conductor with respect to the sun's rays

5.1.3 Convective heat loss, q_c

A bus conductor loses heat through natural or forced convection.

5.1.3.1 Natural convective heat loss

Natural convective heat loss is a function of

a) The temperature difference between the conductor surface and the ambient air temperature
b) The orientation of the conductor's surface
c) The width of the conductor's surface
d) The conductor's surface area

5.1.3.2 Forced convective heat loss

Forced convective heat loss is a function of

a) The temperature difference between the conductor's surface and the ambient air temperature
b) The length of flow path over the conductor
c) The wind speed
d) The conductor's surface area

5.1.4 Radiation heat loss, q_r

A conductor loses heat through the emission of radiated heat. The heat lost is a function of

a) The difference in the absolute temperature of the conductor and surrounding bodies
b) The emissivity of the conductor's surface
c) The conductor's surface area

5.1.5 Conductive heat loss, q_{cond}

Conduction is a minor method of heat transfer since the contact surface is usually very small. Conduction may cause an increase in the temperature of the equipment attached to the bus conductor. Conductive heat loss is usually neglected in bus-ampacity calculations.

5.2 Conductor temperature limits

5.2.1 Continuous (see IEEE Std C37.30-1997)

Aluminum alloy and copper conductors may be operated continuously at 90 °C without appreciable loss of strength. They may also be operated at 100 °C under emergency conditions with some annealing. Copper may, however, suffer excessive oxidation if operated at or above 80 °C. Conductors should not be operated at temperatures high enough to damage the connected equipment.

5.2.2 Fault conditions (see ANSI C37.32-1996)

A conductor's temperature will rise rapidly under fault conditions. This is due to the inability of the conductor to dissipate the heat as rapidly as it is generated. Annealing of conductor alloys may occur rapidly at these elevated temperatures. The maximum fault current that can be allowed for copper and aluminum alloy conductors may be calculated using Equations (3) and (4). In general, the final temperature of the conductor is limited to the maximum temperature considered for thermal expansion (see Clause 13).

For aluminum conductors [40% to 65% International Annealed Copper Standard (IACS) conductivity],

$$I = C \times 10^6 A \sqrt{\frac{1}{t} \log_{10} \frac{T_f - 20 + (15150/G)}{T_i - 20 + (15150/G)}} \tag{3}$$

where

C = 92.9 for Metric units [.144 for English units]
I = maximum allowable root-mean-square (rms) value of fault current, A
A = conductor cross-sectional area, mm^2 [in^2]
G = conductivity in percent International Annealed Copper Standard (IACS)
t = duration of fault, s
T_f = allowable final conductor temperature, °C
T_i = conductor temperature at fault initiation, °C

And for copper conductors [95% to 100% International Annealed Copper Standard (IACS) conductivity],

$$I = C \times 10^6 A \sqrt{\frac{1}{t} \log_{10} \frac{T_f - 20 + (25400/G)}{T_i - 20 + (25400/G)}} \tag{4}$$

C = 142 for Metric units [0.22 for English units]

All other variables have been defined previously.

5.2.3 Attached equipment

Since heat generated in the bus conductor may be conducted to attached equipment, allowable conductor temperatures may be governed by the temperature limitations of attached equipment. Equipment temperature limitations should be obtained from the applicable specification or the manufacturer. High-voltage air switches and bus supports are described in IEEE Std C37.30-1997.

5.3 Ampacity tables

The ampacities for most aluminum-alloy and copper bus-conductor shapes are included in Annex B. These ampacities were calculated using the methods outlined in Annex C, which neglect conductive heat loss.

6. Corona and radio influence

Corona develops when the voltage gradient at the surface of a conductor exceeds the dielectric strength of the air surrounding the conductor and ionizes the air molecules. Radio influence (RI) is caused by corona. In practice, corona has not been a factor in rigid-bus design at 115 kV and below. However, the rigid-bus designer should be aware that radio influence can be produced at any voltage by arcing due to poor bonding between bus conductors and associated hardware.

The proximity and largeness of the equipment within a substation create multiple low-impedance paths to ground for radio-frequency current. The Radio Noise Subcommittee of the IEEE Transmission and Distribution Committee states that actual radio influence will be less than that calculated because of this effect [B15].[7]

The designer's problem is to select a bus conductor and specify bus hardware that is corona free during fair-weather conditions at the operating voltage, altitude, and temperature. It should be noted that corona may exist under wet or contaminated conditions.

[7]The numbers in brackets preceded by the letter B correspond to those of the bibliography in Clause 14.

6.1 Conductor selection

For corona-free operation, the maximum surface voltage gradient of the bus-conductor E_m should be less than the allowable surface voltage gradient E_o.

Four basic factors determine the maximum surface voltage gradient of a smooth bus-conductor E_m. They are

1) Conductor diameter or shape
2) Distance from ground
3) Phase spacing
4) Applied voltage

Circular bus shapes will generally give the best performance. A smooth surface condition is important if operating near the allowable surface voltage gradient.

Formulae are provided in Annex D for calculating the maximum surface voltage gradient for a smooth, circular bus-conductor E_m. The calculation should be 110% of the nominal line-to-ground voltage to provide for an operating margin.

The allowable surface voltage gradient for equal radio-influence generation E_o for smooth, circular bus conductors is a function of bus diameter, barometric pressure, and operating temperature. Annex D gives a method for determining the allowable surface voltage gradient.

6.2 Hardware specifications

Bus fittings and hardware for use in rigid-bus structures should be specified as being free of corona under fair-weather conditions at the intended operating voltage, altitude, and temperature.

It should be noted that the testing methods referred to in 6.2.1 do not require the control of air temperature and air pressure during testing. The specifier should refer to Annex D to determine the difference between the allowable voltage gradients under expected operating conditions and possible laboratory conditions. If the difference is significant, the designer may specify that the testing voltage be increased according to the methods of Annex D to compensate for the test pressure and temperature.

6.2.1 Testing methods

Bus fittings and hardware should be tested by the manufacturer in a laboratory under simulated field configuration. All bus fittings and hardware should be tested while attached to a section of the bus conductor for which they are to be used.

6.2.1.1 Visual corona

The visual corona extinction voltage should be tested according to NEMA CC 1-1993.

6.2.1.2 Radio-influence voltage (RIV) level

The radio-influence voltage (RIV) level should be tested according to NEMA 107-1988.

6.2.2 Acceptance criteria

The following performance should be specified for fittings and hardware under fair-weather conditions.

6.2.2.1 Visual corona

The extinction voltage for visual corona should be at least 110% of nominal operating voltage or at least 110% of the testing voltage adjusted to compensate for pressure and temperature.

6.2.2.2 Radio-influence voltage (RIV)

The specified radio-influence voltage (RIV) limits for various bus system components should match those given in the following standards:

a) For fittings and connectors, see NEMA CC 1-1993.
b) For insulators and hardware assemblies, see ANSI C29.9-1983.

7. Conductor vibration

A span of rigid conductor has its own natural frequency of vibration. If the conductor is displaced from its equilibrium position and released, it will begin to vibrate at this natural frequency. The magnitude of the oscillations will decay due to damping. If, however, the conductor is subjected to a periodic force whose frequency is near the natural frequency of the span, the bus may continue to vibrate and the amplitude will increase.

This vibration may cause damage to the bus conductor by fatigue or by excessive fiber stress.

7.1 Natural frequency

The natural frequency of a conductor span is dependent upon the manner in which the ends are supported and upon the conductor's length, mass, and stiffness. The natural frequency of a conductor span can be calculated using Equation (5).

$$f_{\mathrm{b}} = \frac{\pi K^2}{CL^2}\sqrt{\frac{EJ}{m}} \tag{5}$$

where

C = 20 for Metric units [24 for English units]
f_{b} = natural frequency of conductor span, Hz
L = span length, m [ft]
E = modulus of elasticity, MPa [lbf/in^2]
J = moment of inertia of cross-sectional area, cm^4 [in^4]
m = mass per unit length, kg/m [lbf/ft]
K = 1.00 for two pinned ends (dimensionless)
K = 1.22 for one pinned end and one fixed end (dimensionless)
K = 1.51 for two fixed ends (dimensionless)

End conditions can range between fixed and pinned. A fixed end is not free to rotate (moment resisting), whereas a pinned end is free to rotate (not moment resisting). Because of structure flexibility and connection friction, the end conditions are not truly fixed or pinned. However, the end conditions are generally closer to fixed than to pinned.

7.2 Driving functions

Either alternating current or wind may induce vibrations in a bus conductor with frequencies near the natural frequency of the bus conductor.

7.2.1 Current-induced vibrations

Currents flowing through parallel conductors create magnetic fields that interact and exert forces on the parallel conductors. This driving force oscillates at twice the power frequency.

If the calculated natural frequency of a bus span is found to be greater than half the current-force frequency (that is, greater than the power frequency), the bus spans' calculated natural frequency should be changed or a dynamic analysis should be made to determine stresses involved.

7.2.2 Wind-induced vibration

When a laminar (constant, nonturbulent) wind flows across a conductor, aeolian vibration may occur. This vibration may cause bus-conductor fatigue. Laminar flow does not usually occur at high wind speeds because of the ground effects created by terrain, trees, buildings, local thermal conditions, etc. Experience has shown that wind with speeds up to 15 mi/h can have laminar flow.

The maximum frequency of the aeolian force for circular conductors may be calculated from Equation (6), which is based on the Strovhal formula.

$$f_a = \frac{CV}{d} \tag{6}$$

where

C = 5.15 for Metric units [3.26 for English units]
f_a = maximum aeolian force frequency, Hz
V = maximum wind speed for laminar flow, km/h [mi/h]
d = conductor diameter in, [cm]

Formulae for calculating aeolian force frequency for bus cross-sectional shapes other than circular are not available.

If twice the calculated natural frequency of the bus span is greater than the aeolian force frequency, then the bus span length should be changed or the bus should be damped.

7.3 Damping

Bus spans may be damped to reduce aeolian vibration. For tubular bus conductors, damping may be accomplished by installing stranded bare cable inside the bus conductor to dissipate vibrational energy. The cable should be of the same material as the bus conductor to prevent corrosion, and the weight of the cable should be from 10% to 33% of the bus-conductor weight, although some designers have found that from 3% to 5% of the bus-conductor weight is adequate. In some locations, the audible noise generated by stranded cable dampers may be unacceptable.

Commercially available vibration dampers may be used for both tubular and nontubular conductors. Commercial vibration dampers should be sized and placed according to the manufacturer's recommendations.

8. Conductor gravitational forces

Gravitational forces determine the vertical deflection of bus conductors and are a component of the total force, which the conductor must withstand. Gravitational forces consist of the weights of the conductor, damping material, ice, and concentrated masses.

8.1 Conductor

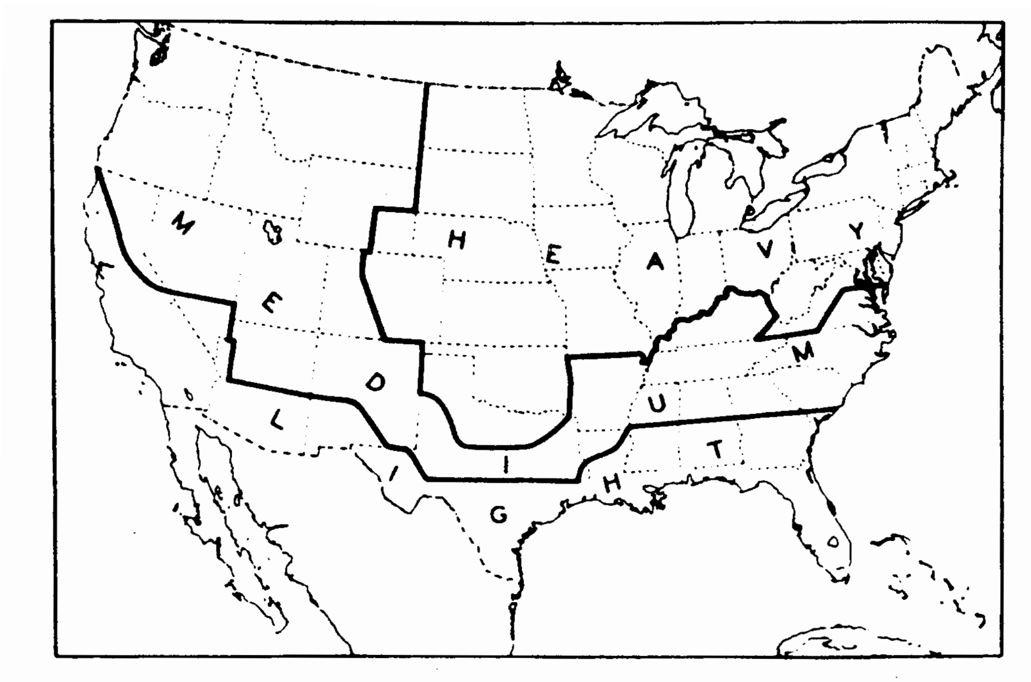
Conductor weight should be obtained from applicable specifications or from the manufacturer.

8.2 Damping material

The weight of the material used to damp vibration should be included in computing gravitational forces. If commercial dampers are used, these should be considered as concentrated masses.

8.3 Ice

The minimum radial ice thickness used for design should be determined from IEEE Std C2-1997. See Figure 2 or [B3].



Loading	Radial Ice Thickness
Heavy	1.27 cm [0.5 in]
Medium	0.64 cm [0.25 in]
Light	0.00 cm [0.0 in]

Figure 2—General loading map showing territorial division of the United States with respect to loading of overhead lines

Consideration should be given to special local conditions where greater ice thicknesses may occur, such as near a cooling-tower installation. The ice weight on a circular conductor is given as

$$F_I = C\pi W_I r_I (d + r_I) \tag{7}$$

where

C = 0.0001 for Metric units [12 for English units]
F_I = ice unit weight, N/m [lbf/ft]
W_I = ice weight = 7.18, N/m^3 [0.0330, lbf/in^3]
r_I = radial ice thickness, cm [in]
d = outside conductor diameter, cm [in]

Equation (7) may be simplified to

$$F_I = Cr_I(d + r_I) \tag{8}$$

where

$C = 2.26 \times 10^{-3}$ for Metric units [1.24 for English units]

Similar equations may be derived for other conductor shapes.

8.4 Concentrated masses

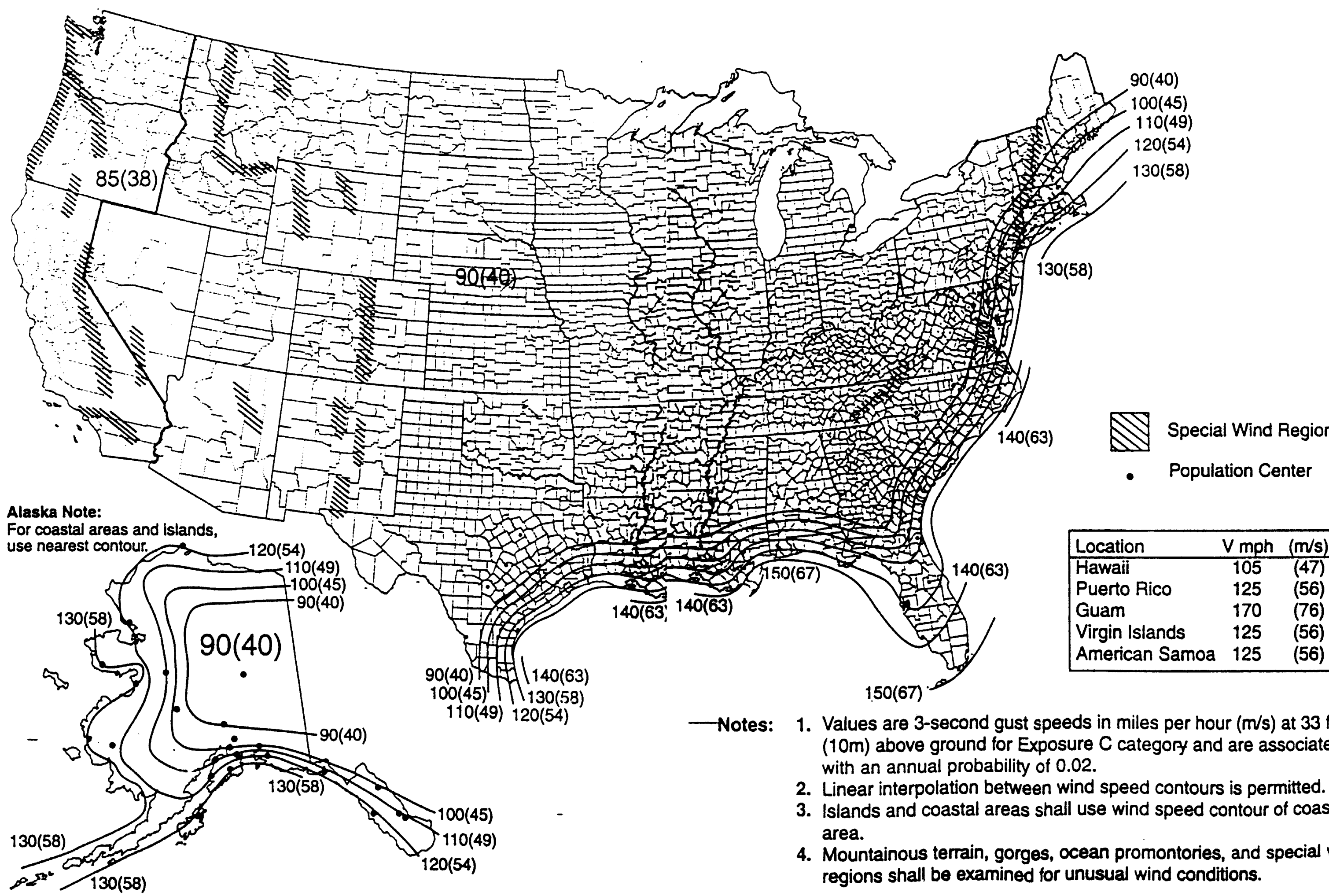
Gravitational forces due to concentrated masses (vibration dampers, equipment attachments, cross conductors, etc.) should be determined and included in the summation of gravitational forces.

9. Conductor wind forces

The bus structure should be capable of withstanding the mechanical forces due to expected winds. The maximum force due to wind may occur either during extreme wind conditions with no ice or high wind conditions with ice. In general, the maximum wind speed with ice is less than the extreme wind speed.

The annual extreme fastest-mile wind speeds for design without ice should be determined from ASCE 7-95. See Figure 3 or [B3]. The choice of the 50- or 100-year recurrence map depends upon the degree of hazard to life or property. Local or state codes should be followed if their wind-force requirements exceed those determined by reference to ASCE 7-95.

The fastest-mile wind speed with ice should be determined from the ice/wind history at the substation site. In general, the wind speed that occurs after icing conditions is lower than the annual extreme fastest-mile wind speed.



Location	V mph	(m/s)
Hawaii	105	(47)
Puerto Rico	125	(56)
Guam	170	(76)
Virgin Islands	125	(56)
American Samoa	125	(56)

Notes:

1. Values are 3-second gust speeds in miles per hour (m/s) at 33 ft (10m) above ground for Exposure C category and are associated with an annual probability of 0.02.
2. Linear interpolation between wind speed contours is permitted.
3. Islands and coastal areas shall use wind speed contour of coastal area.
4. Mountainous terrain, gorges, ocean promontories, and special wind regions shall be examined for unusual wind conditions.

From ASCE 7-95
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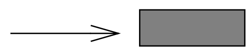
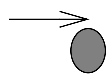
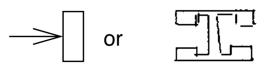
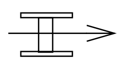
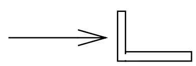
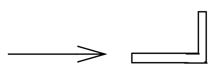
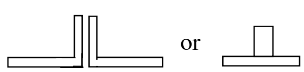
Figure 3—Basic wind speed—miles per hour (mi/h) annual extreme fastest-mile speed 33 ft (10 m) above ground, 50 yr mean recurrence interval

Table 1 displays the drag coefficients for structural shapes in relation to profile and wind direction.

Table 1—Drag coefficients for structural shapes

Profile and Wind Direction	C_D
	2.03
	1.00
	2.00
	2.04
	2.00
	1.83
	1.99

Factors that will affect wind forces are the speed and gust of the wind, radial ice thickness, and the shape, diameter, height, and exposure of the conductors.

The unit wind force for bus is given as

$$F_W = C\, C_D K_Z G_F V^2\, I(d + 2r_I) \tag{9}$$

where

$C = 6.13 \times 10^{-3}$ for Metric unit [2.132×10^{-4} for English units]
F_W = wind unit force on bus, N/m t [lbf/f]
d = outside conductor diameter, cm [in]
r_I = radial ice thickness, cm [in]

C_D = drag coefficient, (see 9.1)
K_Z = height and exposure factor, (see 9.2)
G_F = gust factor, (see 9.3)
V = wind speed at 9.1 m (30 ft) above ground, km/h [mi/h]
I = importance factor (see 9.4)

9.1 Drag coefficient, C_D

The wind force exerted on a conductor varies with the shape of the conductor. This variation is reflected in the drag coefficient C_D. The drag coefficient for smooth tubular conductors is 1.0. Coefficients for other shapes are given in Table 1.

9.2 Height and exposure factor, K_Z

In the height zone from 0 m (0 ft) to 9.1 m (30 ft) and for exposure category A, B, C, the height and exposure factor K_z = 1.0, and the wind speed at 9.1 m (30 ft) should be used. For exposure category, K_z = 1.16. ASCE 7-95 has a detailed definition of each of these exposure categories. Summarized definitions are as follows:

— *Exposure A:* Large city centers with at least 50% of the buildings having a height in excess of 21.3 m (70 ft).
— *Exposure B:* Urban and suburban areas, wooded areas, or other terrain with numerous closely spaced obstructions having the size of single-family dwellings or larger.
— *Exposure C:* Open terrain with scattered obstructions having heights generally less than 9.1 m (30 ft). This category includes flat open country and grassland.
— *Exposure D:* Flat, unobstructed areas exposed to wind flowing over open water for a distance of at least 1.61 km (1 mi).

9.3 Gust factors, G_F

A gust factor G_F of 0.8 shall be used for exposure A and B, and 0.85 shall be used for exposure C and D.

9.4 Importance factor, I

The importance factor I for electric substations shall be 1.15 as classified by ASCE 7-95.

10. Conductor fault current forces

The magnetic fields produced by fault current cause forces on the bus conductors. The bus conductors and bus supports must be strong enough to withstand these forces.

The force imparted to the bus structure by fault current is dependent on conductor spacing, magnitude of fault current, type of short circuit, and degree of short-circuit asymmetry. Other factors to be considered are support flexibility, and corner and end effects.

10.1 Classical equation

The classical equation for the force between parallel, infinitely long conductors in a flat configuration due to an asymmetrical short-circuit current is as follows:

For Metric units:

$$F_{SC} = \frac{5.4\Gamma(2\sqrt{2}I_{SC})^2}{10^7(D)} = \frac{43.2\Gamma I_{SC}^2}{10^7(D)} \tag{10a}$$

For English units:

$$F_{SC} = \frac{2\Gamma(2\sqrt{2}I_{SC})^2}{10^4(D)} = \frac{1.6\Gamma I_{SC}^2}{10^4(D)} \tag{10b}$$

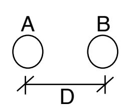
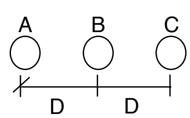
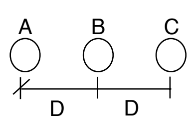
where

F_{sc} = Fault current unit force, N/m [lbf/ft]
I_{sc} = symmetrical rms fault current, A
D = conductor spacing center-to-center, cm [in]
Γ = constant based on type of fault and conductor location (see Table 2)

Equation (10) assumes that the fault is initiated to produce the maximum current offset. The magnitudes of the fault current I_{SC} for each type of fault (three-phase, phase-to-phase, etc.) are not equal to each other and will depend upon the electrical system parameters.

Unless data on the present and future available fault currents are known, it is suggested that the interrupting capability of the substation interrupting equipment (circuit breakers, circuit switchers, etc.) be considered as the maximum I_{SC}.

Table 2—Constant Γ for calculating short-circuit current forces

Type of short circuit	Configuration	Force on conductor	Γ
Phase-to-Phase		A or B	1.00
Three-Phase		B	0.866
Three-Phase		A or C	0.808

10.2 Decrement factor

Due to the presence of system impedance, there is a decrement of the asymmetrical wave in the first half-cycle of the fault. Therefore, it is practical to assume a lower value of peak fault current. Using a value of 1.6 as the assumed current offset, Equation (10) becomes

$$F_{SC} = \frac{C\Gamma(D_f\sqrt{2}I_{SC})^2}{(D)} \tag{11}$$

where

$C = 0.2 \times 10\ 10^{-4}$ for Metric units [5.4×10^{-7} for English units]
F_{sc} = short-circuit current unit force, N/mt [lbf/f]
I_{sc} = symmetrical short-circuit current, A, rms
D = conductor spacing center-to-center, cm [in]
Γ = constant based on type of short circuit and conductor location (see Table 2)
D_f = decrement factor is given by the following equation:

$$D_f = \sqrt{1 + \frac{T_a}{t_f}\left(1 - \exp^{-\frac{2t_f}{T_a}}\right)} \tag{11a}$$

where

$T_a = \frac{X}{R}\frac{1}{2\pi f}$ (X = System Reactance, R = System Resistance, and f = 60Hz)

t_f = fault current duration in seconds

If a system's maximum current offset is less than the assumed value of 1.6, the force will be further reduced.

Equation (11) gives the maximum force in the first half-cycle of the fault. The actual force present when maximum conductor span deflection occurs is usually less because

a) Most conductor spans will not reach maximum deflection until after the first quarter-cycle, and
b) Additional current decrement occurs as the fault continues.

The combination of these two factors results in lower maximum deflection than the deflection caused by a steady-state force equal to the maximum force in the first quarter-cycle.

Tests have shown that conductor spans with natural frequencies of 1/10 of the power frequency or less, and in a system with an X/R ratio of 13 or less, will have fault current forces of less than one half the calculated first quarter-cycle force when the conductor span reaches full deflection.

In practice, a static force equal to the first quarter-cycle force is generally used to calculate rigid-bus structure deflections and stresses. This practice has given a margin of safety to the rigid-bus structure design for fault current forces.

10.3 Mounting-structure flexibility

Because of their flexibility, the bus and mounting structures are capable of absorbing energy during a fault. Thus, depending on the type of mounting structures and their heights, the effective fault current forces can be further reduced by using Equation (12).

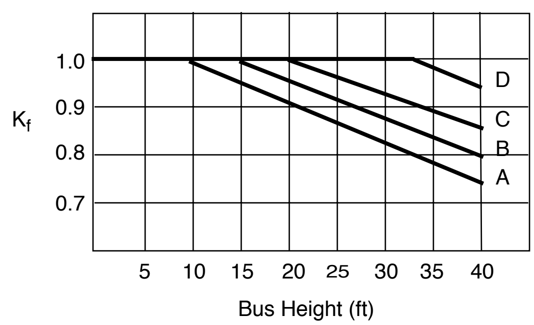
$$F_{SC} = K_f \frac{C\Gamma(D_f\sqrt{2}I_{SC})^2}{(D)} \tag{12}$$

K_f = mounting-structure flexibility factor

Values of K_f, as suggested by Working Group D3 for single-phase mounting structures, are given in Figure 4. K_f is usually assumed to be unity for three-phase mounting structures.

All other variables have been defined previously.

There have been fault current tests conducted on specific combinations of rigid-bus structures with mounting structures that indicate lower values of K_f than those shown in Figure 4. Where the structures are similar to those tested, the lower values of K_f may apply. Future work is expected to produce methods for determining values of K_f for specific mounting structures.



A = Lattice and tubular aluminum
B = Tubular and wide-flange steel, and wood pole
C = Lattice steel
D = Solid concrete

Figure 4—K_f for various types of single-phase mounting structures

10.4 Corner and end effects

The values for the short-circuit current force calculated by Equations (10), (11), and (12) are for parallel and infinitely long conductors. The results for short bus lengths will be conservative because of end effects. The equations cannot be used for special cases, such as corners and nonparallel conductors. Annex E provides methods for determining the forces for special bus configurations.

11. Conductor strength considerations

Any span of a bus conductor must have enough stiffness and strength to withstand the expected forces of gravity, wind, and short circuits, and maintain its mechanical and electrical integrity. The span should also not sag excessively under normal conditions.

This clause only includes equations for single-level, single-span bus conductors supported at both ends, and for continuous bus conductors supported at equal spans without concentrated loads. Annex F of this guide covers analysis for other forms. The simple static method given in Annex F can be used for analyzing distributed loads and concentrated loads on continuous bus conductors supported at equal or unequal spans.

This method is particularly valuable for analyzing a two-level bus arrangement, where one bus at the lower level supports the other bus at upper level using an A-frame form. In this case, the forces acting on the upper bus are transmitted to the lower bus as concentrated loads at each base of the A-frame. Such loading can impose severe stress on the bus conductors and the supporting insulators. A full static analysis could be performed to determine the stresses at various points using the method described in Annex F or other methods obtained from structural design handbooks.

11.1 Vertical deflection

11.1.1 Vertical deflection limits

The allowable vertical deflection of a bus conductor is usually limited by appearance. Commonly used limits are based either on the ratio of conductor deflection to span length (1 : 300 to 1 : 150), or the vertical dimension of the conductor (0.5% to 1% times the vertical dimension). Vertical deflection depends upon the total gravitational force. In practice, since appearance is usually not considered during icing conditions, the ice weight is usually not considered for vertical deflection. However, if the vertical deflection during icing conditions is important, then ice weight should be considered.

11.1.2 Total gravitational force

The total gravitational force on a conductor is the sum of the weights of the conductor, ice, damping material, and any concentrated loads. Without concentrated loads,

$$F_G = F_c + F_I + F_D \tag{13}$$

where

F_G = total bus unit weight, N/m [lbf/ft]
F_c = conductor unit weight, N/m [lbf/ft]
F_I = ice unit weight, N/m [lbf/ft]
F_D = clamping material unit weight, N/m [lbf/ft]

If the bus span is subjected to concentrated loads, the force distribution on that span should be analyzed more thoroughly.

11.1.3 Allowable span length for vertical deflection

The maximum allowable bus span length may be calculated with a given vertical deflection limit, end conditions, and total vertical force distribution.

The deflection may be based on either the vertical conductor dimension or a fraction of the span length.

End conditions for a single span range between fixed and pinned—A fixed end is not free to rotate (moment resisting), whereas a pinned end is free to rotate (not moment resisting). In reality, because of supporting structure flexibility and connection friction, the end conditions are not truly fixed or pinned.

If the end conditions of the single bus span are unknown, then Equation (14) for two pinned ends should be used. For a continuous bus, end conditions are assumed to be pinned and mid-supports are fixed.

11.1.3.1 Single-span bus, two pinned ends

For a single span with two pinned ends, the allowable span length based on vertical deflection may be calculated by one of the equations given in Table 3:

Table 3—Modulus of elasticity E for common conductor alloys

Bus-Conductor Alloy	*E*	
	kPa	lbf/in^2
Aluminum 6061-T6	6.895×10^7	10×10^6
Aluminum 6063-T6	6.895×10^7	10×10^6
Aluminum 6101-T61	6.895×10^7	10×10^6
Copper	11.03×10^7	16×10^6

$$L_D = C\sqrt[4]{\frac{384(E)(J)(Y_A)}{5F_G}} \text{ or } L_D = C\left[\frac{384(E)(J)(Y_A)}{5F_G}\right]^{\frac{1}{4}} \quad (14)$$

where

C = 1.78 for Metric units [1.86 for English units]

L_D = allowable span length, cm [in]

Y_A = allowable deflection, cm [in]

E = modulus of elasticity, kPa [lbf/in^2] (see Table 3)

J = cross-sectional moment of inertia, cm^4 [in^4] (see [B1], chapter 13)

F_G = total bus unit weight, N/m [lbf/ft]

or

$$L_D = C\sqrt[3]{\frac{384(E)(J)(Y_B)}{5F_G}} \text{ or } L_D = C\left[\frac{384(E)(J)(Y_B)}{5F_G}\right]^{\frac{1}{3}} \quad (15)$$

where

Y_B = allowable deflection as a fraction of span length

All other variables have been defined previously.

11.1.3.2 Single-span bus, two fixed ends

For a span with two fixed ends, the allowable span length based on vertical deflection may be calculated by Equations (16) or (17).

$$L_D = C\sqrt[4]{\frac{384(E)(J)(Y_A)}{F_G}} \text{ or } L_D = C\left[\frac{384(E)(J)(Y_A)}{F_G}\right]^{\frac{1}{4}} \quad (16)$$

or

$$L_D = C\sqrt[3]{\frac{384(E)(J)(Y_B)}{F_G}} \text{ or } L_D = C\left[\frac{384(E)(J)(Y_B)}{F_G}\right]^{\frac{1}{3}} \quad (17)$$

where

C = 1.78 for Metric units [1.86 for English units]

L_D = allowable span length, cm [in]

Y_A = allowable deflection, cm [in]

Y_B = allowable deflection as a fraction of span length

E = modulus of elasticity, kPa [lbf/in^2] (see Table 3)

J = cross-sectional moment of inertia, cm^4 [in^4] (see [B1], chapter 13)

F_G = total bus unit weight, N/m [lbf/ft]

11.1.3.3 Single-span bus, one pinned end, one fixed end

For a single span with one pinned end and one fixed end, Equations (18) and (19) may be used to calculate the maximum allowable span length based on vertical deflection.

$$L_D = C\sqrt[4]{\frac{185(E)(J)(Y_A)}{F_G}} \text{ or } L_D = C\left[\frac{185(E)(J)(Y_A)}{F_G}\right]^{\frac{1}{4}} \quad (18)$$

or

$$L_D = C\sqrt[3]{\frac{185(E)(J)(Y_B)}{F_G}} \text{ or } L_D = C\left[\frac{185(E)(J)(Y_B)}{F_G}\right]^{\frac{1}{3}} \quad (19)$$

where

C = 1.78 for Metric units [1.86 for English units]

All other variables have been defined previously.

11.1.3.4 Continuous bus

For a continuous bus, Equations (20) or (21) may be used to calculate the maximum allowable span length based on vertical deflection.

$$L_D = C\sqrt[4]{\frac{185(E)(J)(Y_A)}{F_G}} \text{ or } L_D = C\left[\frac{185(E)(J)(Y_A)}{F_G}\right]^{\frac{1}{4}} \quad (20)$$

$$L_D = C\sqrt[3]{\frac{185(E)(J)(Y_B)}{F_G}} \text{ or } L_D = C\left[\frac{185(E)(J)(Y_B)}{F_G}\right]^{\frac{1}{3}} \quad (21)$$

where

C = 1.78 for Metric units [1.86 for English units]

All other variables have been defined previously.

NOTE—The above equations are for two-span buses. For continuous bus of more than two spans, the maximum deflection occurs in the end spans and is slightly less than that of the two-span bus. The allowable span will be slightly longer.

11.2 Conductor fiber stress

In some cases, span lengths may be limited by the fiber stress of the bus-conductor material. The elastic limit and minimum yield stresses for common conductor materials are tabulated in Table 4. In practice, when wind and gravitational forces are combined, the elastic limit stress is commonly used as the maximum allowable stress. When wind, gravitational, and fault current forces F_{SC} are combined, the minimum yield stress is commonly used as the maximum allowable stress, since F_{SC} is conservative.

11.2.1 Effects of welding

Where welded fittings are used for bus, the allowable stress for the bus should be reduced to allow for annealing due to welding. Tests have shown that the reduction in allowable stress is approximately 50% for aluminum. The reduction in allowable stress for copper is dependent on the welding method (brazing, exothermic, etc.) and should be discussed with the manufacturers. Locating the weld in a region of moderate stress is a usual method of offsetting the effect of weld annealing. Where welded splices are used with a tubular bus, the reduction in allowable stress may not be required if a reinforcing insert is incorporated.

11.2.2 Summation of conductor forces

The maximum bending stresses a conductor withstands are a function of the total vectorial force on the conductor. The total force on a conductor in a horizontal configuration is

$$F_T = \sqrt{[(F_w + F_{SC})^2 + (F_G)^2]} \quad (22)$$

where

F_T = total unit force, N/m [lbf/ft]
F_w = wind unit force, N/m [lbf/ft]
F_{SC} = fault unit force, N/m [lbf/ft]
F_G = total bus unit weight, N/m [lb/ft]

Table 4—Allowable stress for common conductor materials

Bus-Conductor Material	Stress (lbf/in^2)		Stress (kPa)	
	Elastic Limit	Minimum Yield	Elastic Limit	Minimum Yield
Aluminum alloy—6063-T6 or 6101-T6	20 500	25 000[a]	141 348	172 375[a]
Aluminum alloy— 6061-T6	29 500	35 000[a]	203 403	241 325[a]
Aluminum alloy— 6061-T61	11 000	15 000[a]	75 845	103 452[a]
Copper No. 110 hard drawn	—	24 000[b]	—	275 800[b]

[a]With 0.2 offset per ASTM B241/B241M-96.
[b]With 0.5% offset per ASTM B188-96.

The angle of the total force below horizontal is

$$\theta = \tan^{-1}\left[\frac{F_G}{F_w + F_{SC}}\right] \tag{23}$$

The total force on a conductor in a vertical configuration is

$$F_T = \sqrt{(F_w)^2 + (F_G + F_{SC})^2} \tag{24}$$

where

F_T = total unit force, lbf/ft [N/m]
F_W = wind unit force, N/m [lbf/ft]
F_G = total bus unit weight, N/m [lbf/ft]
F_{SC} = short-circuit unit force, N/m [lbf/ ft]

The angle of the force below horizontal is

$$\theta = \tan^{-1}\left[\frac{F_G + F_{SC}}{F_w}\right] \tag{25}$$

11.2.3 Allowable span length for fiber stress

The maximum allowable span length for fiber stress may be calculated for any given conductor, total force, and allowable stress. If the conductor cross section is not symmetrical about the direction of the total force, calculations should be made for the conductor section modulus in the direction of the total force.

If the end conditions of the bus span are unknown, then Equation (26) should be used.

11.2.3.1 Two pinned ends

For a single span with two pinned ends, the allowable span length is calculated with Equation 26.

$$L_S = C\sqrt{\frac{8F_A S}{F_T}} \tag{26}$$

where

L_S = maximum allowable length, cm [in]
C = 3.16 for Metric units [3.46 for English units]
F_A = maximum allowable stress, kPa^2 [lbf/in]
S = section modulus, cm^3 [in^3]
F_T = total force, N/m [lbf/ft]

The maximum bending moment will occur at the middle of the span.

11.2.3.2 Single-span bus, two fixed ends

For a span with two fixed ends, the allowable span-length equation based on fiber stress is:

$$L_S = C\sqrt{\frac{12(F_A)(S)}{F_T}} \tag{27}$$

where

C = 3.16 for Metric units [3.46 for English units]

All other variables have been defined previously.

11.2.3.3 Single-span bus, one pinned end, one fixed end

For a single span with one pinned end and one fixed end, the maximum allowable span based on fiber stress may be calculated as follows:

$$L_S = C\sqrt{\frac{8(F_A)(S)}{F_T}} \tag{28}$$

where

C = 3.16 for Metric units [3.46 for English units]

All other variables have been defined previously.

The maximum bending moment will occur at the fixed end of the span.

11.2.3.4 Continuous-span bus

Equation (29a), Equation (29b), and Equation (29c), as follows, may be used to calculate the maximum allowable span length based on fiber stress for a different number of spans.

Number of Spans	Equation	Equation Number
Two-Span Bus	$L_S = C\sqrt{\frac{8(F_A)(S)}{F_T}}$ C = 3.16 Metric units [3.46 for English units]	(29a)
Three-Span Bus	$L_S = C\sqrt{\frac{10(F_A)(S)}{F_T}}$ C = 3.16 Metric units [3.46 for English units]	(29b)
Four-Span Bus	$L_S = C\sqrt{\frac{28(F_A)(S)}{F_T}}$ C = 3.16 Metric units [3.46 for English units]	(29c)

NOTE—The allowable span length is limited by the maximum fiber stress that occurs at the second support from each end. Equation (29c) can be used conservatively for continuous bus with more than a four-span length. L_S, F_A, F_T, and S are as defined earlier.

11.3 Maximum allowable span length

The maximum allowable span length L_A is equal to L_S or L_D, whichever is shorter.

12. Insulator strength considerations

Since the forces on the bus conductors are transmitted to the insulators, the strength of the insulators must be considered. With various bus configurations, insulators may be required to withstand cantilever, compressive, tensile, and torsional forces. Only cantilever forces have been considered in this guide. However, other forces (tension, torsion, and compression) may be critical, requiring consideration in the design.

12.1 Insulator cantilever forces

The insulator cantilever force for some common bus arrangements can be given as a function of the effective conductor span length supported by the insulator and the external forces on the bus and insulator. The external forces are

a) The fault current force on the bus
b) The wind force on the bus and insulator
c) The gravitational forces on the bus, insulator or concentrated masses, or both

The effective conductor span length L_E depends on the span length and the bus-support conditions. Use Table 5 to find L_E for each particular support condition and the number of spans. If the support conditions are not known, then take the support condition that yields the maximum span length L_S calculated in Clause 11.

Table 5—Maximum effective bus span length L_E supported by insulator for common bus arrangements[a]

Bus Configuration	Support Conditions					Maximum Span Length L_E
	S1	S2	S3	S4	S5	
Single-Span	P	P				(1/2)*L*
Single-Span	P	F				(5/8)*L* (Max at S2)
Single-Span	F	F				(1/2)*L*
Two Cont.-Span	P	C	P			(5/4)*L* (Max at S2)
Two Cont.-Span	P	F	F			(9/8)*L* (Max at S2)
Two Cont.-Span	F	F	F			*L* (Max at S2)
Three Cont.-Span	P	C	C	P		11/10 *L* (Max at S2)
Four Cont.-Span	P	C	C	C	P	32/28 *L* (Max at S2

[a]*L* = Bus Span Length - Equal Spans for two or more spans.
L_E = Maximum Effective Span Length.
P = Pinned Support
F = Fixed Support
C = Mid-Support of Continuous Span

NOTE—This table is applicable only to equal span bus arrangement. The mid-support of a continuous bus has only reaction force, but no moment although the continuous bus conductor has a moment at the support point. See Annex F for the method of calculating insulator cantilever forces for individual support of all continuous-bus arrangements. For continuous spans of more than the spans shown, use the equation for the largest span shown for the same end conditions.

12.1.1 Bus short-circuit current force

The short-circuit current force transmitted to the bus-support fitting can be calculated using Equation (30).

$$F_{SB} = L_E F_{SC} \tag{30}$$

where

F_{SB} = bus fault current force transmitted to bus-support fitting, N [lbf]
L_E = effective bus span length, m [ft] (See Table 5)
F_{SC} = fault current unit force as calculated in Clause 10, N/m [lbf/ft]

If the end conditions are unknown, then the fixed end conditions at the bus-support fitting in question and pinned end conditions at the opposite ends of the adjacent spans will yield the maximum effective bus span length. The adjacent bus span lengths $L1$ and $L2$ should be equal to or less than the maximum allowable span length L_D calculated in Clause 11.

12.1.2 Bus wind force

The unit wind force associated with the bus span is the same as that described in Clause 9. The wind force transmitted to the bus-support fitting can be calculated using Equation (31).

$$F_{WB} = L_E F_W \tag{31}$$

where

F_{WB} = bus wind force transmitted to bus-support fitting, N [lbf]

L_E = effective bus span length, m [ft] (see Table 5)

F_W = wind unit force on the bus, N/m [lbf/ft]

12.1.3 Insulator wind force

The wind force on the bus-support insulator is a function of

a) The insulator dimensions
b) The wind speed
c) The gust factor
d) The radial ice thickness
e) The mounting height
f) Exposure to wind

The wind force acting on the center of an insulator can be calculated using Equation (32).

$$F_{WI} = C\, C_D K_Z G_F V^2 (D_i + 2\, r_I)\, H_i \tag{32}$$

where

F_{WI} = wind force on insulator, N [lbf]

C = 6.13 × 10-3 for Metric units [2.132 × 10^{-4} for English]

C_D = drag coefficient

K_Z = height and exposure factor

G_F = gust factor

V = wind speed at 30 ft [9.1m] above ground, km/h [mi/h]

D_i = effective insulator diameter, cm [in]

r_I = radial ice thickness, cm [in]

H_i = insulator height, cm [in] (see Fig 5)

r_I, K_Z, G_F, and V are the same factors used for the wind force on the bus conductor (see Clause 9). C_D is usually considered as unity.

The effective insulator diameter D_i is usually considered as the insulator diameter over the skirts. For tapered insulators the effective diameter is the average diameter and can be calculated using Equation (33).

$$D_i = \frac{D_1 + D_2 + ...D_n}{n} \tag{33}$$

where

D_1, D_2, and D_n = outside diameters of each subassembly for the 1st, 2nd, and nth sections of the insulator (see Figure 5).

The total wind force F_{WI} on a uniform-diameter insulator acts at the center of the insulator (see Figure 5). For a tapered insulator, the total wind force is usually considered acting at the center $H_i/2$ since the resulting error is of small magnitude and is conservative.

12.1.4 Gravitational forces

In some rigid-bus structure configurations, the insulator may be subjected to cantilever gravitational forces. These forces should be added vectorially to the fault current and wind forces. These gravitational forces will be due to the mass of the supported rigid bus, the mass of the insulator itself or other concentrated masses, or both.

The effective weight of the bus mass transmitted to the bus-support fitting can be determined using Equation (34).

$$F_{GB} = L_E F_G \tag{34}$$

where

F_{GB} = effective weight of bus transmitted to bus-support fitting, N [lbf]
L_E = effective bus span length, m [ft] (see Table 5)
F_G = total bus unit weight, N/m [lbf/ft]

If the bus span is subjected to concentrated loads, the force transmitted to the bus-support fitting should be analyzed more thoroughly.

The weight of the insulator F_{GI} should be included in the total cantilever force if the insulator is not mounted vertically.

12.1.5 Total insulator cantilever load

The total cantilever load on an insulator is the summation of the cantilever forces acting on the insulator multiplied by their overload factors.

The total cantilever load on a vertically-mounted insulator supporting a horizontal bus (see Figure 5) can be calculated using Equation (35).

$$F_{IS} = K_1\left[\frac{F_{WI}}{2} + \frac{(H_i + H_f)F_{WB}}{H_i}\right] + K_2\left[\frac{(H_i + H_f)F_{SB}}{H_i}\right] \tag{35}$$

where

F_{IS} = total cantilever load acting at end of insulator, N [lbf]
F_{WI} = wind force on the insulator, N [lbf]
F_{SB} = short-circuit current force transmitted to bus-support fitting, N [lbf]
F_{WB} = bus wind force transmitted to the bus-support fitting, N [lbf]
H_i = insulator height, cm [in]
H_f = bus centerline height above the insulator, cm [in]
K_1 = overload factor applied to wind forces
K_2 = overload factor applied to short-circuit current forces

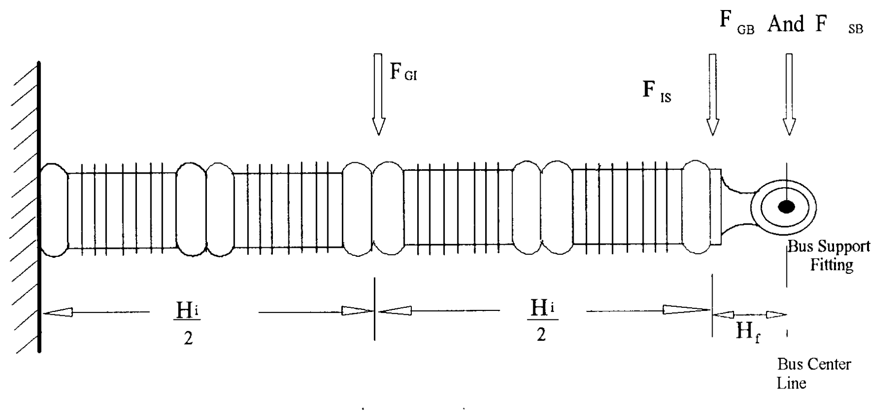


Figure 5—Vertically mounted insulator cantilever forces

The total cantilever load on a horizontally mounted insulator with a horizontal bus (see Figure 6) can be calculated using Equation (36).

$$F_{IS} = K_3\left[\frac{F_{GI}}{2} + \frac{(H_i + H_f)F_{GB}}{H_i}\right] + K_2\left[\frac{(H_i + H_f)F_{SB}}{H_i}\right] \quad (36)$$

where

F_{IS} = total cantilever load acting at end of insulator, N [lbf]
F_{GI} = weight of insulator, N [lbf]
F_{GB} = effective weight of bus transmitted to bus-support fitting, N [lbf]
F_{SB} = short-circuit current force transmitted to bus-support fitting, N [lbf]
H_i = insulator height, cm [in]
H_f = bus centerline distance beyond insulator, cm [in]
K_2 = overload factor applied to fault current forces
K_3 = overload factor applied to gravitational forces

Equations (34), (35), and (36) cover the most common bus and insulator configurations. The designer should examine each configuration to ensure the proper summation of forces acting on the insulator.

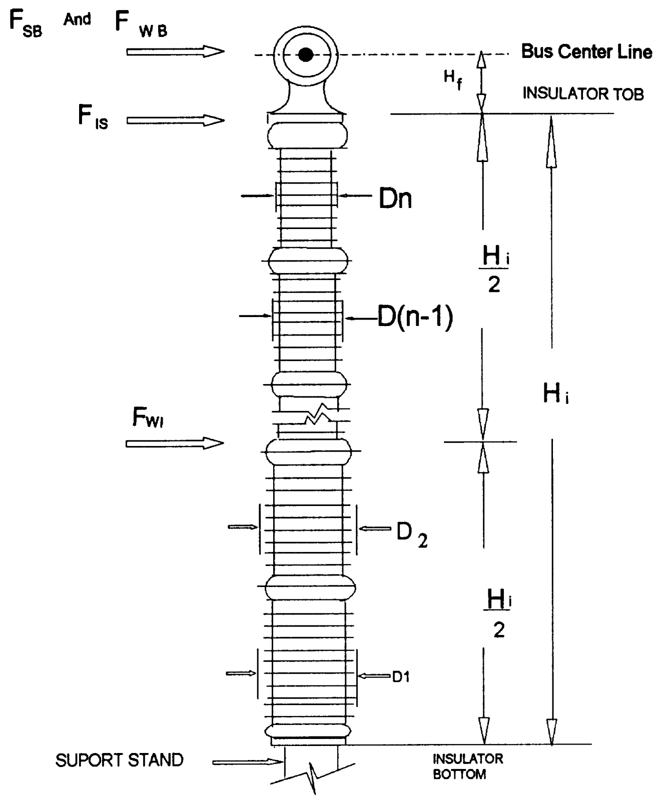


Figure 6—Horizontally mounted insulator cantilever forces

12.2 Insulator force overload factors

Porcelain, unlike metal, is very brittle. The yield and tensile strengths of porcelain have identical values. Because porcelain cannot yield without cracking, an overload factor should be applied to the loads on the insulator.

A conservative value of 2.5 is recommended for overload factors K_1 and K_3 (wind and gravitational forces) by some US insulator manufacturers.

The value of overload factor K_2 (fault current forces) depends upon the natural frequencies of the insulator, of the insulator/mounting structure combination, and of the conductor span. Since the force F_{SC} is conservative, a value of 1.0 can be used for K_2 if

a) The natural frequency of the insulator, together with the effective weight of the conductor span f_i, is less than one half the short-circuit current-force frequency, that is

$$f_i < \frac{120}{2} \text{ Hz for a 60 Hz system} \tag{37}$$

where

f_i = natural frequency of insulator with effective weight of conductor span, Hz

b) The natural frequencies of the insulator/mounting structure combination f_{s1} and f_{s2} and the natural frequency of the conductor span f_b differ by a factor of at least two, that is

$$\frac{f_{s1}}{f_b} < \frac{1}{2} \text{or} \frac{f_{s1}}{f_b} > 2 \tag{37a}$$

$$\frac{f_{s2}}{f_b} < \frac{1}{2} \text{or} \frac{f_{s2}}{f_b} > 2 \tag{37b}$$

where

f_{s1} = first natural frequency of insulator/mounting structure combination, Hz
f_{s2} = second natural frequency of insulator/mounting structure combination, Hz
f_b = natural frequency of the conductor span, Hz

If either of these conditions is not satisfied, a dynamic study should be made to determine an appropriate overload factor, or an overload factor of 2.5 should be used.

The natural frequency of the insulator together with the effective weight of the conductor span can be calculated using Equation (38).

$$f_i = \frac{1}{2\pi}\sqrt{\frac{K_i g}{0.226F_{GI} + F_{GB}}} \tag{38}$$

where

f_i = natural frequency of insulator with effective weight of conductor span, Hz
K_i = insulator cantilever spring constant, N/m [lbf/in]
g = gravitational constant, 9.81m/s^2 [386 in/s^2]
F_{GI} = weight of insulator, N [lbf]
F_{GB} = effective weight of bus transmitted to bus-support fitting, N [lbf]

The natural frequencies of the insulator/mounting structure combination f_{s1} and f_{s2} can be calculated using Equations (39) and (40).

$$f_{s1} = \frac{1}{2\pi}\sqrt{\frac{K_i + K_s}{2m_1} + \frac{K_i}{2m_2} - \frac{1}{2}\sqrt{\left(\frac{K_i + K_s}{m_1} + \frac{K_i}{m_2}\right)^2 - \frac{4K_iK_s}{m_1m_2}}} \tag{39}$$

$$f_{s2} = \frac{1}{2\pi}\sqrt{\frac{K_i + K_s}{2m_1} + \frac{K_i}{2m_2} - \frac{1}{2}\sqrt{\left(\frac{K_i + K_s}{m_1} + \frac{K_i}{m_2}\right)^2 - \frac{4K_iK_s}{m_1m_2}}} \tag{40}$$

where

F_{s1} = first natural frequency of insulator/mounting structure combination, Hz
F_{s2} = second natural frequency of insulator/mounting structure combination, Hz
K_i = insulator cantilever spring constant, N/m [lbf/in]
K_s = mounting structure cantilever spring constant, N/m [lbf/in]

$$m_1 = \frac{0.333F_{GS} + 0.5F_{GI}}{g}$$

$$m_2 = \frac{F_{GB} + 0.226F_{GS}}{g}$$

where

F_{GS} = weight of mounting structure, N [lbf]
F_{GI} = weight of insulator, N [lbf]
F_{GB} = weight of bus, N [lbf]
g = gravitational constant, 9.81/s^2 [386 in/s^2]

The cantilever spring constant for the insulator can be obtained from insulator manufacturers. The cantilever spring constant for a single-phase mounting structure with a constant cross section can be calculated using Equation (41).

$$K_S = C\frac{3EJ}{H_s^3} \quad (41)$$

where

K_S = support cantilever spring constant, N/m [lbf/in]
C = 0.01 for Metric units [1 for English units]
E = modulus of elasticity, N/m [lbf/in^2]
J = cross-sectional moment of inertia, cm^4 [in^4]
H_s = mounting structure length, cm [in]

12.3 Minimum insulator cantilever strength

The minimum published insulator cantilever strength required is

$$S_I \geq F_{IS} \quad (42)$$

where

S_I = minimum published insulator cantilever strength, N [lbf]
F_{IS} = total cantilever load acting at end of insulator, N [lbf]

13. Conductor thermal expansion considerations

When the temperature of a bus conductor is changed, a corresponding change in length results. This change in length can be calculated as

$$\Delta L = \frac{\alpha L_i(T_f - T_i)}{1 + \alpha T_i} \quad (43)$$

where

ΔL = change in span length, m [ft]
α = coefficient of thermal expansion, 1/°C
T_i = initial installation temperature, °C
T_f = final temperature, °C
L_i = span length at the initial temperature, m [ft]

13.1 Thermal loads

If the ends of the conductor are fixed, preventing expansion or contraction, and the conductor temperature is changed, compressive or tensile forces will result. These forces can be computed as

$$F_{TE} = CAE\frac{\Delta L}{L_1} = CAE\alpha(T_i - T_f) \quad (44)$$

where

F_{TE} = thermal force, N [lbf]
C = 0.1 for Metric units [1 for English units]
A = cross-sectional area of the conductor, cm^2 [in^2]
E = modulus of elasticity, kPa [lbf/in^2]
ΔL = change in span length, m [ft]
L_i = span length at the initial temperature, m [ft]
α = coefficient of thermal expansion, 1/°C
T_i = initial installation temperature, °C
T_f = final temperature, °C

The force calculated using Equation (44) does not consider the flexibility of mounting structures or bus structure. Since this flexibility will allow some expansion or contraction of the bus conductor, the forces experienced will be less than the force calculated above.

13.2 Expansion fittings

Since the thermal forces exerted on the bus conductor are independent of span length, provisions should be made for expansion in any bus-conductor span. These provisions may be made with expansion fittings for long buses, or by considering deflection of a bus conductor, bus-conductor bends, insulators, or mounting structures for short buses.

14. Bibliography

[B1] *Aluminum Electrical Conductor Handbook,* Aluminum Company of America, 1989.

[B2] Bates, A. C., "Basic concepts in the design of electric bus for short-circuit conditions," *AIEE Transactions*, no. 57–717, vol. 77, pp. 29–39, Apr. 1958.

[B3] Chaine, P. M., Verge, R. W., Caston-Guay, G., and Gariepy, J., "Wind and ice loading in Canada," *Industrial Meteorology-Study II,* Toronto: Environment Canada, 1974.

[B4] Hartzog, D., *Mechanical Vibrations,* New York: McGraw-Hill, 1956.

[B5] Jacobsen, L. S., and Ayre, R. S., *Engineering Vibrations,* New York: McGraw-Hill, 1958.

[B6] Killian, C. D., “Forces due to short-circuit currents,” *Delta-Star Magazine,* 1943.

[B7] Milton, R. M., and Chambers, F., “Behavior of high-voltage buses and insulators during short circuits,” *AIEE Transactions,* Part 3, vol.74, pp. 742–749, Aug. 1955.

[B8] Morse, P. M., *Vibration and Sound,* New York: McGraw-Hill, 1948.

[B9] Radio Noise Subcommittee of the Transmission and Distribution Committee of the IEEE Power Group. “Radio noise design guide for high-voltage transmission lines.” *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-90, no. 2, pp. 833–842, Mar./Apr. 1971.

[B10] Palante, G., “Study and conclusions from the results of the enquiry on the thermal and dynamic effects of heavy short-circuit currents in high-voltage substations,” *Electra,* no. 12, pp. 51–89, Mar. 1970.

[B11] Pinkham, T. A., and Killeen, N. D., “Short-circuit forces on station post insulators,” *IEEE Paper Number 71TP 40-PWR,* vol. 90, pp. 1688–1697, 1971.

[B12] Schwartz, S. J., “Substation design shows need for bus damping,” *Electrical World,* June 24, 1963.

[B13] Taylor, D. W., and Steuhler, C. M., “Short-circuit forces on 138 kV buses,” *AIEE Transactions,* Part 3, vol. 75, pp. 739–747, Aug. 1956.

[B14] Tompkins, Merrill, and Jones, “Relationships in vibration damping,” *AIEE Transactions,* Part 3, vol. 75, pp. 879–896, Oct. 1956.

[B15] “Transmission system radio influence,” Radio Noise Subcommittee of the Transmission and Distribution Committee of the IEEE Power Group, *Transactions on Power Apparatus and Systems,* Aug. 1965.

[B16] Wilson, W., *The Calculation and Design of Electrical Apparatus,* London: Chapman and Hill, Ltd., 1941.

[B17] “Wind forces on structures,” *Transaction Paper Number 3269-1961,* vol. 126.

Annex A

(informative)

Letter symbols for quantities

Symbol	Meaning
A	cross-sectional area, cm^2 [in^2]
C	temperature, °C
C_D	drag coefficient
D	conductor spacing, center-to-center, cm [in]
D_i	effective insulator diameter, cm [in]
d	conductor outside diameter, cm [in]
E	modulus of elasticity kPA [lbf/in^2]
F	temperature, °F
F	skin-effect coefficient
F_A	maximum allowable stress, Nm^2 [lbf/in^2]
F_c	conductor unit weight, Nm [lbf/ft]
F_D	damping material unit weight, Nm [lbf/ft]
F_G	total bus unit weight, Nm [lbf/ft]
F_{GB}	effective weight of bus transmitted to bus-support fitting, N [lbf]
F_{GI}	weight of insulator, N [lbf]
F_{GS}	weight of mounting structure, N [lbf]
F_I	ice unit weight, Nm [lbf/ft]
F_{is}	total cantilever load acting at end of insulator, N [lbf]
F_{SB}	short-circuit current-force transmitted to bus-support fitting, N [lbf]
F_{SC}	fault current unit force, Nm [lbf/ft]
F_T	total unit force on the bus, Nm [lbf/ft]
F_{TE}	thermal force, N [lbf]
F_W	wind unit force on the bus, Nm [lbf/ft]
F_{WB}	bus wind force transmitted to bus-support fitting, N [lbf]
F_{WI}	wind force on insulator, N [lbf]
f_a	maximum aeolian vibration frequency, Hz
f_b	natural frequency of bus span, Hz
f_i	natural frequency of insulator together with effective weight of bus span, Hz
f_{s1}, f_{s2}	natural frequencies of insulator together with mounting structure, Hz
G	conductivity, %IACS
g	gravitational constant
G_F	gust factor
H_f	bus centerline distance above top of insulator, cm [in]
H_i	insulator height, cm [in]
H_s	mounting structure height, cm [in]
I	current, A, rms
I_{sc}	symmetrical short-circuit current, A, rms
J	moment of inertia of cross-sectional area, cm^4 [in^4]
K	constant used in span natural frequency calculation and dependent upon end conditions
K_f	mounting structure flexibility factor
K_i	insulator cantilever spring constant, Nm [lbf/in]
K_s	mounting structure cantilever spring constant, Nm [lbf/in]
K_Z	height and exposure factor
K_1,K_2,K_3	insulator overload factors

L	span length, m [ft]
L_A	maximum allowable bus span length, m [ft]
L_D	maximum allowable bus span length based on vertical deflection, cm [in]
L_E	effective bus span length, m [ft]
L_i	span length at initial temperature T_i, m [ft]
L_S	maximum allowable bus span length based on fiber stress, cm [in]
L_1,L_2	adjacent bus span lengths, m [ft]
lbf	pound force [N]
lbf	pound mass [N]
m	mass per unit length, kg/m [lbm/ft]
q_c	convective heat loss, W/m [W/ft]
q_{cond}	conductive heat loss, W/m [W/ft]
q_r	radiation heat loss, W/m [W/ft]
q_s	solar heat gain, W/m [W/ft]
R	conductor direct-current resistance, S/m [S/ft]
r_I	radial ice thickness, w/m [in]
S	section modulus, cm^3 [in^3]
S_I	minimum published insulator cantilever strength, N [lbf]
T_f	final conductor temperature, °C
T_i	initial conductor temperature, °C
t	time, s
V	wind speed, km/h [mi/h]
W_I	ice weight, N/cm^3 [lbf/in^3]
Y_A	maximum allowable deflection, in [cm]
Y_B	maximum allowable deflection as a fraction of span length
α	coefficient of thermal expansion
ΔL	L change in span length, m [ft]
θ	angle of total force below horizontal, degrees
λ	ratio of span length to vertical dimension of bus conductor
Γ	multiplying factor based on type of short-circuit current

Annex B

(informative)

Bus-conductor ampacity

The bus-ampacity data included in this annex have been taken from Thermal Consideration for Outdoor Bus-Conductor Design Ampacity Tables, Substation Committee of the IEEE Power Engineering Society.[8]

Table B.1—Single aluminum rectangular bar AC ampacity, with sun (55.0% conductivity)

	Emissivity = 0.20 Temperature Rise Above 40˚C Ambient							Emissivity = 0.50 Temperature Rise Above 40˚C Ambient						
Size (in)	30	40	50	60	70	90	110	30	40	50	60	70	90	110
0.250 by 4.000	1130	1298	1441	1566	1678	1872	2039	1200	1394	1560	1707	1839	2073	2278
0.250 by 5.000	1320	1517	1685	1833	1965	2195	2393	1413	1644	1841	2016	2174	2455	2703
0.250 by 6.000	1497	1723	1915	2084	2235	2500	2729	1615	1881	2109	2311	2495	2821	3110
0.375 by 4.000	1385	1593	1769	1924	2063	2304	2510	1464	1704	1909	2091	2254	2544	2799
0.375 by 5.000	1608	1851	2057	2239	2401	2686	2931	1714	1998	2241	2456	2651	2997	3302
0.375 by 6.000	1815	2091	2326	2533	2718	3044	3326	1950	2275	2554	2801	3026	3426	3782
0.375 by 8.000	2202	2540	2829	3084	3313	3718	4070	2395	2800	3148	3458	3740	4247	4700
0.500 by 4.000	1589	1829	2034	2213	2374	2654	2895	1672	1951	2189	2399	2590	2926	3223
0.500 by 5.000	1835	2115	2353	2562	2750	3079	3364	1949	2276	2556	2805	3030	3430	3785
0.500 by 6.000	2071	2388	2659	2897	3111	3487	3814	2216	2590	2912	3197	3456	3918	4330
0.500 by 8.000	2511	2899	3231	3524	3788	4255	4662	2721	3186	3587	3943	4268	4851	5374
0.625 by 4.000	1776	2047	2277	2479	2660	2977	3249	1861	2177	2446	2683	2898	3278	3614
0.625 by 5.000	2034	2347	2613	2847	3058	3427	3747	2152	2519	2833	3111	3363	3812	4210
0.625 by 6.000	2286	2639	2940	3206	3445	3865	4231	2437	2855	3213	3531	3820	4337	4798
0.625 by 8.000	2760	3190	3558	3884	4177	4696	5151	2982	3498	3942	4337	4698	5347	5929
0.625 by 10.000	3190	3690	4120	4501	4845	5457	5996	3483	4091	4615	5084	5513	6238	6987
0.625 by 12.000	3560	4123	4608	5039	5430	6126	6744	3924	4615	5212	5748	6240	7131	7941
0.750 by 4.000	1935	2232	2486	2708	2907	3256	3557	2021	2368	2664	2926	3163	3582	3953
0.750 by 5.000	2216	2559	2851	3108	3340	3746	4098	2336	2740	3085	3391	3668	4162	4601
0.750 by 6.000	2472	2856	3184	3474	3735	4195	4597	2627	3083	3474	3821	4137	4702	5207
0.750 by 8.000	2984	3452	3852	4207	4527	5094	5592	3214	3776	4260	4691	5085	5793	6430
0.750 by 10.000	3518	4072	4548	4969	5350	6026	6622	3832	4505	5086	5605	6079	6935	7708
0.750 by 12.000	3875	4491	5021	5492	5919	6682	7359	4260	5015	5669	6255	6793	7768	8655

[8]Published by IEEE Transaction on Power Apparatus and Systems, Vol PAS-96, NO 4, July/August 1977.

Table B.2—Single aluminum rectangular bar AC ampacity, without sun (55.0% conductivity)

	Emissivity = 0.20 Temperature Rise Above 40˚C Ambient							Emissivity = 0.50 Temperature Rise Above 40˚C Ambient						
Size (in)	**30**	**40**	**50**	**60**	**70**	**90**	**110**	**30**	**40**	**50**	**60**	**70**	**90**	**110**
0.250 by 4.000	1158	1322	1462	1585	1695	1887	2052	1265	1449	1608	1749	1877	2105	2306
0.250 by 5.000	1354	1546	1711	1856	1986	2213	2409	1492	1710	1899	2068	2221	2494	2737
0.250 by 6.000	1538	1757	1945	2111	2260	2521	2747	1708	1959	2177	2372	2549	2867	3150
0.375 by 4.000	1423	1625	1798	1950	2086	2324	2528	1553	1780	1975	2149	2308	2589	2838
0.375 by 5.000	1654	1890	2092	2270	2429	2710	2952	1821	2087	2319	2526	2714	3050	3349
0.375 by 6.000	1869	2136	2366	2569	2751	3072	3350	2073	2378	2644	2882	3099	3488	3835
0.375 by 8.000	2271	2598	2880	3130	3355	3753	4102	2552	2931	3262	3560	3833	4324	4767
0.500 by 4.000	1638	1871	2070	2246	2403	2679	2917	1786	2047	2273	2474	2657	2984	3273
0.500 by 5.000	1893	2164	2396	2601	2786	3109	3390	2082	2388	2654	2892	3109	3497	3843
0.500 by 6.000	2137	2444	2708	2941	3152	3522	3844	2369	2719	3024	3297	3546	3995	4396
0.500 by 8.000	2595	2970	3294	3580	3840	4298	4701	2912	3347	3726	4068	4381	4946	5457
0.625 by 4.000	1836	2098	2322	2520	2697	3008	3277	2002	2295	2549	2775	2981	3349	3675
0.625 by 5.000	2104	2406	2665	2894	3100	3463	3778	2313	2654	2951	3216	3458	3893	4281
0.625 by 6.000	2365	2706	2999	3259	3493	3906	4267	2620	3008	3346	3650	3928	4428	4877
0.625 by 8.000	2859	3274	3632	3949	4237	4747	5196	3206	3686	4106	4484	4831	5459	6027
0.625 by 10.000	3307	3790	4207	4579	4917	5518	6050	3748	4313	4809	5257	5669	6420	7102
0.625 by 12.000	3696	4239	4709	5129	5512	6196	6805	4227	4869	5434	5945	6418	7282	8072
0.750 by 4.000	2006	2293	2539	2756	2951	3293	3589	2188	2509	2787	3035	3262	3666	4026
0.750 by 5.000	2298	2628	2912	3163	3389	3788	4135	2526	2899	3224	3515	3780	4257	4684
0.750 by 6.000	2564	2934	3253	3535	3791	4243	4639	2838	3260	3628	3959	4262	4808	5299
0.750 by 8.000	3097	3548	3937	4283	4596	5153	5644	3472	3992	4448	4859	5237	5921	6542
0.750 by 10.000	3655	4188	4649	5060	5433	6097	6684	4140	4763	5311	5805	6260	7088	7841
0.750 by 12.000	4030	4622	5136	5595	6013	6762	7429	4605	5305	5921	6480	6996	7940	8804

Table B.3—Aluminum tubular bus — Schedule 40 AC ampacity (53.0% conductivity)

SPS[a] Size	OD	Wall Thickness	Emissivity = 0.20, With Sun Temperature Rise Above 40˚C Ambient							Emissivity = 0.20, Without Sun Temperature Rise Above 40˚C Ambient						
(in)	(in)	(in)	30	40	50	60	70	90	110	30	40	50	60	70	90	110
1.0	1.315	0.133	591	688	770	840	903	1011	1102	638	728	804	871	931	1035	1123
1.5	1.900	0.145	837	978	1097	1199	1290	1447	1580	914	1043	1153	1250	1336	1486	1614
2.0	2.375	0.154	1035	1213	1362	1490	1605	1802	1969	1139	1300	1438	1558	1666	1854	2015
2.5	2.875	0.203	1377	1618	1818	1992	2147	2413	2640	1527	1743	1928	2090	2235	2488	2705
3.0	3.500	0.216	1666	1962	2208	2422	2612	2940	3220	1861	2126	2351	2550	2728	3038	3305
3.5	4.000	0.226	1897	2239	2523	2770	2989	3367	3690	2132	2435	2695	2923	3127	3484	3792
4.0	4.500	0.237	2134	2523	2847	3127	3376	3807	4175	2412	2755	3049	3307	3539	3945	4295
5.0	5.563	0.258	2636	3127	3536	3890	4204	4748	5213	3010	3439	3807	4131	4422	4933	5374
6.0	6.625	0.280	3153	3752	4250	4681	5063	5726	6294	3633	4152	4597	4990	5343	5963	6500
8.0	8.625	0.322	4142	4954	5629	6213	6731	7631	8404	4843	5538	6135	6662	7138	7975	8703

SPS[a] Size	OD	Wall Thickness	Emissivity = 0.50, With Sun Temperature Rise Above 40˚C Ambient							Emissivity = 0.50, Without Sun Temperature Rise Above 40˚C Ambient						
(in)	(in)	(in)	30	40	50	60	70	90	110	30	40	50	60	70	90	110
1.0	1.315	0.133	572	690	788	872	948	1078	1190	686	785	870	945	1013	1133	1238
1.5	1.900	0.145	805	981	1127	1252	1363	1556	1723	992	1136	1260	1370	1469	1645	1800
2.0	2.375	0.154	991	1217	1402	1561	1703	1949	2161	1244	1425	1581	1720	1845	2068	2264
2.5	2.875	0.203	1314	1623	1876	2094	2287	2623	2914	1677	1921	2132	2320	2490	2793	3060
3.0	3.500	0.216	1582	1969	2284	2555	2795	3214	3576	2056	2357	2617	2848	3059	3434	3766
3.5	4.000	0.226	1796	2248	2614	2929	3208	3694	4116	2366	2712	3012	3280	3523	3957	4342
4.0	4.500	0.237	2015	2534	2954	3315	3635	4192	4675	2686	3080	3421	3726	4004	4500	4940
5.0	5.563	0.258	2474	3142	3680	4141	4550	5262	5880	3375	3872	4304	4690	5041	5671	6232
6.0	6.625	0.280	2943	3771	4435	5003	5506	6382	7144	4098	4703	5230	5701	6131	6902	7591
8.0	8.625	0.322	3830	4982	5899	6681	7373	8581	9633	5515	6334	7048	7688	8274	9328	10 274

[a]SPS = Standard pipe size

Table B.4—Aluminum tubular bus—Schedule 80 AC ampacity (53.0%) conductivity

Size SPS[a]	OD	Wall Thickness	Emissivity = 0.20, With Sun Temperature Rise Above 40 °C Ambient							Emissivity = 0.20, Without Sun Temperature Rise Above 40 °C Ambient						
			30	40	50	60	70	90	110	30	40	50	60	70	90	110
(in)	(in)	(in)														
1.0	1.315	0.179	672	783	875	956	1027	1149	1253	726	828	915	991	1059	1177	1277
1.5	1.900	0.200	967	1131	1267	1385	1490	1671	1825	1056	1205	1332	1444	1543	1716	1864
2.0	2.375	0.218	1212	1420	1595	1745	1879	2110	2306	1334	1523	1684	1825	1952	2172	2360
2.5	2.875	0.276	1580	1855	2086	2285	2462	2768	3029	1751	1999	2211	2397	2564	2855	3104
3.0	3.500	0.300	1930	2273	2559	2807	3028	3408	3733	2157	2463	2725	2955	3161	3522	3832
3.5	4.000	0.318	2210	2608	2940	3228	3483	3925	4303	2484	2838	3140	3406	3645	4062	4422
4.0	4.500	0.337	2499	2954	3334	3663	3955	4460	4893	2824	3226	3570	3873	4146	4622	5034
5.0	5.563	0.375	3104	3683	4165	4583	4954	5598	6150	3544	4050	4484	4867	5212	5816	6340
6.0	6.625	0.432	3801	4525	5127	5649	6113	6919	7610	4379	5007	5546	6022	6450	7204	7859
8.0	8.625	0.500	4927	5898	6706	7407	8031	9118	10 056	5761	6592	7308	7943	8516	9528	10 413

Size SPS[a]	OD	Wall Thickness	Emissivity = 0.50, With Sun Temperature Rise Above 40 °C Ambient							Emissivity = 0.50, Without Sun Temperature Rise Above 40 °C Ambient						
			30	40	50	60	70	90	110	30	40	50	60	70	90	110
(in)	(in)	(in)														
1.0	1.315	0.179	650	785	896	992	1078	1226	1353	780	893	989	1075	1152	1289	1408
1.5	1.900	0.200	930	1134	1302	1446	1575	1798	1990	1146	1312	1455	1582	1697	1901	2079
2.0	2.375	0.218	1161	1425	1642	1829	1994	2282	2531	1457	1669	1851	2014	2161	2422	2652
2.5	2.875	0.276	1507	1862	2152	2402	2624	3009	3343	1923	2203	2445	2661	2856	3204	3512
3.0	3.500	0.300	1833	2282	2647	2961	3240	3725	4146	2382	2731	3032	3301	3545	3981	4366
3.5	4.000	0.318	2092	2619	3046	3413	3739	4307	4799	2756	3160	3510	3822	4106	4613	5063
4.0	4.500	0.337	2358	2967	3459	3882	4257	4911	5479	3144	3606	4006	4364	4690	5272	5789
5.0	5.563	0.375	2912	3700	4335	4879	5362	6204	6937	3974	4560	5069	5525	5941	6687	7352
6.0	8.625	0.432	3547	4548	5350	6037	6647	7711	8638	4940	5672	6309	6880	7401	8339	9178
8.0	8.625	0.500	4556	5931	7028	7965	8797	10 252	11 526	6561	7541	8396	9166	9871	11 145	12 293

[a]SPS = Standard pipe size

Table B.5—Single aluminum angle bus AC ampacity (55.0% conductivity)

	Emissivity = 0.20, With Sun Temperature Rise Above 40˚C Ambient							Emissivity = 0.20, Without Sun Temperature Rise Above 40˚C Ambient						
Size (in)	30	40	50	60	70	90	110	30	40	50	60	70	90	110
3.250 by 3.250 by 0.250	1588	1857	2083	2279	2454	2757	3016	1734	1980	2191	2376	2542	2831	3081
4.000 by 4.000 by 0.250	1835	2153	2420	2652	2859	3217	3525	2022	2311	2557	2775	2970	3312	3608
4.000 by 4.000 by 0.375	2178	2557	2875	3153	3400	3831	4201	2401	2744	3039	3299	3533	3943	4300
4.500 by 4.500 by 0.375	2343	2757	3104	3408	3678	4150	4558	2597	2970	3291	3574	3829	4279	4670
5.000 by 5.000 by 0.375	2518	2969	3347	3677	3972	4488	4934	2806	3210	3557	3865	4143	4633	5061

	Emissivity = 0.50, With Sun Temperature Rise Above 40˚C Ambient							Emissivity = 0.50, Without Sun Temperature Rise Above 40˚C Ambient						
Size (in)	30	40	50	60	70	90	110	30	40	50	60	70	90	110
3.250 by 3.250 by 0.250	1550	1889	2169	2412	2628	3007	3336	1902	2180	2420	2634	2828	3174	3481
4.000 by 4.000 by 0.250	1786	2194	2530	2821	3080	3535	3931	2236	2564	2848	3102	3334	3747	4114
4.000 by 4.000 by 0.375	2120	2606	3007	3354	3664	4208	4685	2654	3045	3385	3688	3965	4461	4904
4.500 by 4.500 by 0.375	2277	2813	3254	3637	3979	4580	5108	2885	3312	3683	4016	4320	4866	5356
5.000 by 5.000 by 0.375	2443	3032	3516	3936	4311	4973	5555	3130	3595	4000	4363	4696	5295	5833

Table B.6—Double aluminum angle bus AC ampacity (55.0% conductivity)

	Emissivity = 0.20, With Sun Temperature Rise Above 40˚C Ambient							Emissivity = 0.20, Without Sun Temperature Rise Above 40˚C Ambient						
Size (in)	30	40	50	60	70	90	110	30	40	50	60	70	90	110
3.250 by 3.250 by 0.250	2875	3370	3794	4166	4501	5086	5590	3045	3513	3917	4276	4600	5170	5663
4.000 by 4.000 by 0.250	3361	3949	4451	4892	5289	5984	6583	3579	4131	4608	5032	5415	6090	6675
4.000 by 4.000 by 0.375	3952	4646	5240	5764	6236	7065	7784	4208	4860	5426	5929	6385	7191	7893
4.500 by 4.500 by 0.375	4340	5109	5766	6346	6868	7786	8581	4636	5356	5980	6536	7040	7930	8707
5.000 by 5.000 by 0.375	4739	5585	6307	6945	7519	8528	9403	5077	5866	6552	7162	7715	8693	9546

	Emissivity = 0.50, With Sun Temperature Rise Above 40˚C Ambient							Emissivity = 0.50, Without Sun Temperature Rise Above 40˚C Ambient						
Size (in)	30	40	50	60	70	90	110	30	40	50	60	70	90	110
3.250 by 3.250 by 0.250	2832	3407	3893	4318	4700	5370	5953	3247	3749	4187	4578	4933	5566	6122
4.000 by 4.000 by 0.250	3306	3996	4577	5086	5542	6345	7044	3835	4432	4952	5416	5839	6593	7258
4.000 by 4.000 by 0.375	3887	4702	5389	5992	6535	7492	8329	4510	5215	5830	6382	6885	7785	8582
4.500 by 4.500 by 0.375	4265	5173	5938	6609	7213	8277	9209	4983	5764	6446	7057	7615	8614	9499
5.000 by 5.000 by 0.375	4653	5658	6503	7245	7911	9087	10 117	5472	6331	7081	7755	8369	9470	10 447

Table B.7—Aluminum integral web channel bus AC ampacity (55.0% conductivity)

Size	Emissivity = 0.20, With Sun Temperature Rise Above 40 °C Ambient							Emissivity = 0.20, Without Sun Temperature Rise Above 40 °C Ambient						
	30	40	50	60	70	90	110	30	40	50	60	70	90	110
(in)														
4.000 by 4.000 by 0.250	2395	2948	3404	3799	4150	4761	5286	2949	3402	3795	4144	4461	5022	551
4.000 by 4.000 by 0.312	2603	3206	3703	4134	4517	5183	5757	3213	3706	4133	4514	4860	5472	600
6.000 by 4.000 by 0.375	3391	4168	4812	5370	5867	6734	7483	4161	4800	5356	5851	6301	7099	779
6.000 by 5.000 by 0.375	3558	4420	5129	5743	6289	7241	8062	4483	5175	5778	6316	6805	7674	843
6.000 by 6.000 by 0.550	4287	5335	6200	6950	7621	8795	9816	5412	6254	6990	7649	8250	9325	1027
8.000 by 5.000 by 0.500	4617	5695	6588	7365	8058	9272	10 326	5699	6582	7351	8039	8666	9783	1077
8.000 by 8.000 by 0.500	5849	7228	8375	9374	10 271	11 846	13 223	7212	8345	9335	10 224	11 036	12 491	1378
12.000 by 12.000 by 0.625	8610	10 614	12 296	13 774	15 108	17 477	19 574	10 466	12 138	13 608	14 936	16 156	18 361	2034

Size	Emissivity = 0.50, With Sun Temperature Rise Above 40 °C Ambient							Emissivity = 0.50, Without Sun Temperature Rise Above 40 °C Ambient						
	30	40	50	60	70	90	110	30	40	50	60	70	90	110
(in)														
4.000 by 4.000 by 0.250	2463	3102	3627	4081	4486	5198	5819	3208	3707	4143	4535	4894	5538	611
4.000 by 4.000 by 0.312	2677	3376	3948	4444	4887	5665	6345	3497	4041	4517	4944	5336	6039	666
6.000 by 4.000 by 0.375	3572	4470	5211	5856	6434	7453	8349	4568	5280	5905	6467	6982	7911	874
6.000 by 5.000 by 0.375	3718	4722	5544	6256	6893	8014	8999	4929	5701	6379	6990	7551	8563	947
6.000 by 6.000 by 0.550	4403	5646	6661	7540	8329	9722	10 954	5963	6904	7733	8483	9174	10 428	1156
8.000 by 5.000 by 0.500	4886	6145	7185	8091	8906	10 347	11 622	6308	7300	8172	8960	9685	10 999	1218
8.000 by 8.000 by 0.500	5922	7594	8963	10 152	11 219	13 110	14 786	7990	9262	10 384	11 400	12 338	14 044	1559
12.000 by 12.000 by 0.625	8584	11 093	13 153	14 949	16 570	19 463	22 058	11 724	13 621	15 304	16 839	18 264	20 878	2327

Table B.8—Single copper rectangular bar AC ampacity, with sun (99.0% conductivity)

	Emissivity = 0.35 Temperature Rise Above 40˚C Ambient							Emissivity = 0.85 Temperature Rise Above 40˚C Ambient						
Size (in)	30	40	50	60	70	90	110	30	40	50	60	70	90	110
0.250 by 4.000	1516	1751	1951	2127	2286	2564	2806	1661	1948	2194	2412	2611	2965	3281
0.250 by 5.000	1764	2040	2276	2484	2671	3002	3291	1955	2296	2589	2850	3088	3515	3898
0.250 by 6.000	2010	2327	2599	2838	3054	3437	3773	2250	2646	2987	3290	3568	4067	4517
0.375 by 4.000	1824	2112	2356	2572	2766	3107	3405	1985	2337	2638	2906	3149	3584	3973
0.375 by 5.000	2122	2458	2746	3000	3229	3633	3988	2337	2754	3112	3430	3721	4243	4712
0.375 by 6.000	2407	2792	3121	3412	3675	4141	4552	2679	3159	3573	3942	4279	4887	5436
0.375 by 8.000	2934	3409	3816	4178	4505	5089	5608	3319	3922	4442	4908	5335	6109	6813
0.500 by 4.000	2083	2415	2699	2948	3173	3569	3915	2253	2662	3011	3321	3603	4108	4560
0.500 by 5.000	2404	2790	3120	3412	3675	4141	4551	2633	3113	3524	3890	4224	4826	5367
0.500 by 6.000	2717	3156	3532	3865	4166	4701	5174	3007	3558	4031	4453	4839	5536	6167
0.500 by 8.000	3312	3853	4317	4730	5105	5774	6369	3729	4417	5011	5542	6030	6916	7723
0.625 by 4.000	2253	2617	2928	3203	3451	3889	4274	2423	2873	3258	3599	3911	4469	4971
0.625 by 5.000	2619	3045	3409	3731	4023	4540	4996	2854	3384	3840	4245	4615	5282	5885
0.625 by 6.000	2951	3433	3847	4213	4546	5137	5662	3251	3857	4378	4843	5269	6040	6739
0.625 by 8.000	3598	4192	4702	5156	5568	6306	6966	4034	4791	5443	6028	6565	7541	8433
0.625 by 10.000	4179	4875	5474	6009	6496	7372	8158	4752	5648	6424	7121	7763	8936	10 012
0.625 by 12.000	4758	5555	6244	6860	7422	8435	9348	5474	6511	7411	8222	8970	10 339	11 601
0.750 by 4.000	2455	2857	3199	3502	3775	4258	4683	2626	3125	3550	3928	4271	4888	5443
0.750 by 5.000	2834	3300	3699	4051	4370	4937	5438	3073	3656	4155	4599	5005	5737	6398
0.750 by 6.000	3204	3732	4185	4587	4951	5600	6177	3513	4179	4752	5262	5729	6575	7343
0.750 by 8.000	3881	4527	5082	5576	6026	6831	7551	4334	5159	5870	6507	7092	8157	9130
0.750 by 10.000	4509	5265	5917	6498	7029	7982	8840	5109	6085	6929	7687	8386	9662	10 835
0.750 by 12.000	5119	5983	6729	7396	8006	9107	10 100	5869	6995	7971	8850	9661	11 147	12 519

Table B.9—Single copper rectangular bar AC ampacity, without sun (99.0% conductivity)

	Emissivity = 0.35 Temperature Rise Above 40˚C Ambient							Emissivity = 0.85 Temperature Rise Above 40˚C Ambient						
Size (in)	30	40	50	60	70	90	110	30	40	50	60	70	90	110
0.250 by 4.000	1577	1802	1996	2168	2322	2595	2833	1793	2059	2290	2498	2688	3030	3337
0.250 by 5.000	1838	2102	2330	2533	2715	3039	3323	2114	2429	2705	2953	3180	3592	3965
0.250 by 6.000	2098	2401	2663	2896	3107	3481	3811	2436	2801	3121	3410	3675	4157	4595
0.375 by 4.000	1908	2182	2418	2627	2816	3150	3442	2167	2489	2770	3023	3254	3672	4049
0.375 by 5.000	2221	2542	2819	3065	3288	3683	4032	2550	2932	3266	3568	3844	4347	4802
0.375 by 6.000	2522	2889	3206	3488	3744	4199	4603	2924	3364	3751	4099	4421	5006	5539
0.375 by 8.000	3081	3532	3924	4274	4593	5164	5673	3628	4179	4665	5105	5513	6258	6941
0.500 by 4.000	2189	2505	2777	3018	3236	3623	3962	2485	2855	3179	3470	3737	4221	4658
0.500 by 5.000	2527	2894	3211	3493	3749	4204	4606	2900	3335	3717	4062	4379	4956	5480
0.500 by 6.000	2858	3275	3636	3958	4251	4773	5237	3310	3810	4250	4647	5014	5683	6294
0.500 by 8.000	3489	4002	4448	4847	5211	5863	6447	4103	4729	5281	5782	6246	7097	7879
0.625 by 4.000	2379	2724	3021	3286	3526	3953	4330	2700	3104	3458	3778	4071	4604	5088
0.625 by 5.000	2765	3168	3517	3828	4111	4615	5062	3171	3650	4069	4449	4799	5437	6019
0.625 by 6.000	3117	3573	3969	4323	4645	5222	5736	3607	4154	4636	5072	5475	6213	6889
0.625 by 8.000	3804	4365	4854	5291	5691	6411	7057	4469	5153	5757	6307	6816	7752	8615
0.625 by 10.000	4423	5081	5654	6169	6642	7496	8266	5262	6073	6792	7448	8057	9182	10 225
0.625 by 12.000	5042	5795	6454	7046	7591	8578	9473	6060	7000	7835	8597	9308	10 622	11 845
0.750 by 4.000	2605	2983	3310	3601	3865	4335	4750	2956	3400	3789	4139	4462	5049	5582
0.750 by 5.000	3006	3445	3825	4164	4473	5024	5515	3446	3967	4425	4839	5221	5918	6555
0.750 by 6.000	3397	3895	4328	4715	5067	5699	6263	3929	4526	5052	5529	5970	6778	7518
0.750 by 8.000	4117	4726	5256	5732	6167	6951	7655	4834	5575	6231	6827	7381	8399	9340
0.750 by 10.000	4787	5499	6122	6681	7195	8123	8963	5690	6569	7348	8059	8721	9944	11 078
0.750 by 12.000	5439	6253	6965	7607	8198	9269	10 242	6532	7547	8449	9273	10 043	11 468	12 795

Table B.10—Copper tubular bus—Schedule 40 AC ampacity (99.0% conductivity)

Size		Wall	Emissivity = 0.35, With Sun Temperature Rise Above 40˚C Ambient							Emissivity = 0.35, Without Sun Temperature Rise Above 40˚C Ambient						
SPS[a]	OD	Thickness	30	40	50	60	70	90	110	30	40	50	60	70	90	110
(in)	(in)	(in)														
1.0	1.315	0.127	771	912	1029	1131	1220	1375	1506	878	1002	1107	1200	1283	1428	1552
1.5	1.900	0.150	1131	1347	1526	1681	1818	2054	2255	1313	1499	1658	1798	1923	2143	2332
2.0	2.375	0.157	1383	1656	1881	2075	2246	2543	2796	1628	1859	2056	2231	2387	2661	2899
2.5	2.875	0.188	1755	2111	2403	2655	2878	3264	3594	2091	2389	2644	2868	3071	3426	3734
3.0	3.500	0.219	2214	2675	3054	3380	3669	4169	4597	2673	3054	3381	3670	3930	4388	4787
4.0	4.500	0.250	2870	3492	4002	4441	4829	5502	6080	3530	4035	4470	4855	5202	5815	6350
6.0	6.625	0.250	3903	4807	5544	6177	6737	7708	8545	4955	5669	6285	6831	7324	8199	8968
8.0	8.625	0.313	5281	6570	7617	8514	9308	10 687	11 880	6871	7868	8728	9493	10 187	11 422	12 512
Size		Wall	Emissivity = 0.85, Without Sun Temperature Rise Above 40 ˚C Ambient							Emissivity = 0.85, With Sun Temperature Rise Above 40 ˚C Ambient						
SPS[a]	OD	Thickness	30	40	50	60	70	90	110	30	40	50	60	70	90	110
(in)	(in)	(in)														
1.0	1.315	0.127	726	916	1069	1199	1315	1515	1688	978	1120	1243	1353	1452	1629	1786
1.5	1.900	0.150	1054	1354	1593	1797	1977	2289	2559	1482	1698	1886	2054	2206	2479	2722
2.0	2.375	0.157	1279	1665	1970	2230	2458	2855	3200	1851	2122	2358	2569	2762	3106	3414
2.5	2.875	0.188	1611	2123	2526	2867	3168	3689	4142	2394	2747	3054	3328	3579	4030	4433
3.0	3.500	0.219	2014	2692	3221	3668	4062	4745	5339	3083	3539	3936	4292	4618	5204	5729
4.0	4.500	0.250	2579	3517	4242	4852	5389	6321	7131	4112	4723	5256	5736	6175	6968	7682
6.0	6.625	0.250	3425	4848	5925	6827	7617	8988	10 182	5863	6741	7509	8201	8836	9988	11 031
8.0	8.625	0.313	4543	6632	8190	9488	10 624	12 596	14 315	8220	9459	10 545	11 527	12 431	14 075	15 569

[a]SPS = Standard pipe size

Table B.11—Copper tubular bus—Schedule 80 AC ampacity (99.0% conductivity)

Size SPS[a]	OD	Wall Thickness	Emissivity = 0.35, With Sun Temperature Rise Above 40˚C Ambient							Emissivity = 0.35, Without Sun Temperature Rise Above 40˚C Ambient						
			30	40	50	60	70	90	110	30	40	50	60	70	90	110
(in)	(in)	(in)														
1.0	1.315	0.182	903	1069	1206	1325	1430	1611	1765	1029	1174	1297	1406	1503	1673	1818
1.5	1.900	0.203	1289	1536	1741	1917	2073	2343	2573	1498	1710	1891	2051	2194	2445	2661
2.0	2.375	0.221	1610	1928	2190	2416	2616	2962	3258	1895	2164	2395	2598	2780	3100	3377
2.5	2.875	0.280	2093	2517	2866	3168	3434	3896	4292	2493	2848	3153	3422	3664	4089	4459
3.0	3.500	0.304	2536	3065	3501	3876	4209	4785	5279	3062	3500	3876	4209	4508	5036	5497
4.0	4.500	0.341	3256	3963	4543	5043	5486	6255	6917	4004	4580	5075	5513	5910	6610	7224
6.0	6.625	0.437	4789	5906	6820	7606	8306	9525	10 584	6081	6965	7730	8411	9030	10 132	11 108
8.0	8.625	0.500	6076	7571	8790	9841	10 776	12 412	13 842	7906	9066	10 073	10 973	11 794	13 265	14 579
Size SPS[a]	**OD**	**Wall Thickness**	**Emissivity = 0.85, With Sun Temperature Rise Above 40˚C Ambient**							**Emissivity = 0.85, Without Sun Temperature Rise Above 40 ˚C Ambient**						
			30	**40**	**50**	**60**	**70**	**90**	**110**	**30**	**40**	**50**	**60**	**70**	**90**	**110**
(in)	(in)	(in)														
1.0	1.315	0.182	851	1073	1252	1405	1540	1775	1978	1146	1313	1457	1585	1701	1909	2092
1.5	1.900	0.203	1202	1544	1817	2050	2255	2612	2920	1690	1937	2151	2343	2517	2829	3106
2.0	2.375	0.221	1489	1938	2294	2597	2863	3326	3728	2155	2471	2746	2992	3216	3619	3978
2.5	2.875	0.280	1921	2532	3013	3420	3780	4404	4946	2855	3276	3642	3971	4271	4810	5293
3.0	3.500	0.304	2308	3086	3693	4207	4659	5446	6130	3532	4056	4512	4922	5296	5972	6579
4.0	4.500	0.341	2926	3992	4816	5511	6122	7186	8113	4664	5360	5967	6513	7015	7921	8739
6.0	6.625	0.437	4203	5956	7288	8407	9391	11 107	12 612	7195	8282	9236	10 099	10 894	12 343	13 664
8.0	8.625	0.500	5277	7642	9452	10 967	12 300	14 629	16 679	9457	10 899	12 170	13 324	14 391	16 346	18 140

[a]SPS = Standard pipe size

Table B.12—Double copper channel bus AC ampacity (99.0% conductivity)

	Emissivity = 0.35, With Sun Temperature Rise Above 40˚C Ambient							Emissivity = 0.35, Without Sun Temperature Rise Above 40˚C Ambient						
Size (in)	**30**	**40**	**50**	**60**	**70**	**90**	**110**	**30**	**40**	**50**	**60**	**70**	**90**	**110**
3.000 by 1.313 by 0.216	2785	3347	3819	4232	4601	5246	5801	3178	3671	4098	4478	4822	5430	5961
4.000 by 1.750 by 0.240	3697	4470	5118	5684	6190	7075	7841	4283	4951	5531	6048	6517	7348	8076
4.000 by 1.750 by 0.338	4106	4969	5695	6331	6902	7906	8780	4757	5504	6155	6737	7267	8212	9044
5.000 by 2.188 by 0.338	4967	6040	6942	7731	8440	9686	10 772	5827	6746	7548	8266	8920	10 087	11 117
6.000 by 2.688 by 0.384	5932	7235	8332	9293	10 159	11 686	13 025	6995	8107	9079	9953	10 751	12 182	13 453
	Emissivity = 0.85, With Sun Temperature Rise Above 40˚C Ambient							**Emissivity = 0.85, Without Sun Temperature Rise Above 40˚C Ambient**						
Size (in)	**30**	**40**	**50**	**60**	**70**	**90**	**110**	**30**	**40**	**50**	**60**	**70**	**90**	**110**
3.000 by 1.313 by 0.216	2733	3430	4003	4499	4941	5718	6395	3504	4053	4533	4963	5356	6061	6689
4.000 by 1.750 by 0.240	3619	4593	5390	6078	6693	7772	8714	4764	5514	6171	6762	7303	8276	9145
4.000 by 1.750 by 0.338	4019	5106	5998	6771	7464	8685	9759	5290	6129	6867	7532	8143	9248	10 241
5.000 by 2.188 by 0.338	4851	6222	7341	8310	9177	10 706	12 052	6526	7565	8480	9306	10 065	11 440	12 680
6.000 by 2.688 by 0.384	5770	7460	8836	10 029	11 099	12 990	14 663	7888	9154	10 271	11 283	12 217	13 915	15 455

Annex C

(informative)

Thermal considerations for outdoor bus-conductor design

By the Substation Committee of the IEEE Power Engineering Society.[9]

C.1 Abstract

Outdoor rigid bus design is based on several limiting criteria. This paper brings to a single source the thermal considerations of rigid bus design namely, transfer of heat and properties of material. It concerns itself with aluminum alloys, copper and copper alloys and the currently acceptable shapes. Historically thermal designs have been conservative. This paper will allow the engineer to re-examine the factors involved in increased current loadings of rigid bus and possibly determine new thermal limits.

C.2 Introduction

Thermal considerations entering into the design of bus conductors for outdoor substations fall into two general categories, transfer of heat and properties of materials. Each of these subjects will be considered in detail in this paper. The first, transfer of heat to and from the conductor, is relatively independent of the material and is mainly a function of the geometry of the conductor, proximity to other surfaces or conductors, atmospheric conditions, and geographic location. The most important element in the computation is the estimate of forced convection arising from wind currents. A method is given here to compute heat losses due to forced and natural convection and radiation and heat gained from the sun. Using the formulas provided it is possible to calculate the current carrying capacity of any conductor corresponding to a given temperature rise. Examples are provided showing methods for calculating the ampacity of conventional types of bus conductors, e.g., bar, tube, channel, angle, integral web, etc.

The second subject, properties of materials, includes the effects of temperature and outdoor exposure on the mechanical strength, electrical resistivity, dimensional stability, and surface condition of the conductor. Aluminum alloys, copper, and copper alloys are included in the discussion and tabulations. No attempt has been made to consider the relative merits of the conductors. Instead, technical information is provided which must be coupled with economic factors when optimizing design and selecting materials.

C.3 Heat transfer

Usually well over half the heat generated by resistance losses in a bus conductor is removed from the surface by convection of the surrounding air. The remainder is given off by radiation from external surfaces. Unfortunately, it is not at all convenient to run controlled outdoor tests to determine the appropriate heat transfer coefficients. As a result there is very little independent support for the formulas found in the literature.

A variety of formulas can be found for the sizes of conductors of interest. All show that convective heat transfer out-of-doors exceeds that in the indoors when it is assumed that the wind velocity is 2 feet per second (fps). However, the difference between the indoor and outdoor rating is often not very great. If a slower

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wind velocity is assumed, the outdoor heat losses may be calculated as lower than those indoors. This is not plausible. It is therefore, concluded that assumption of a 2 fps wind is a conservative, yet realistic approach, and it will be used in the examples given herein.

The difference between indoor and outdoor convection losses are found to diminish with increasing conductor size and increasing temperature rise. This is because an increase in the temperature rise leads to natural drafts which can be as effective as a slight breeze in promoting heat transfer. Similarly, with large conductors, the assumed 2 fps wind speed is so low as to add very little benefit over natural convection.

For the purpose of calculating ampacity, conditions which are least advantageous for convection must be considered. Thus, it is assumed that there is only a 2 fps wind. (See note 1 following the references of this annex) It is to be expected that when the flow is at an angle or normal to the surface, heat transfer will increase. Likewise, it is wise to stipulate that the emissivity is a low value when there is no solar heating. This will provide the most conservative ampacity rating. In contrast, when there can be considerable solar heating a high value of emissivity essentially equal to solar absorptivity may give the most conservative ampacity rating.

In connection with this last point, it should be noted that solar heating of the conductor always diminishes ampacity and can result in outdoor current ratings which are lower than indoor ratings. This is less likely on smaller conductors for which forced (outdoor) convective heat transfer coefficients are relatively high. However, for large conductors with high absorptivity, the heat gain from solar radiation can exceed the improvement in convective heat transfer due to the wind effect and ratings are reduced accordingly.

C.3.1 Assumptions

Some assumptions will be made about the properties of air in order to reduce the number of terms which must be carried through the computations. These approximations will have negligible effect on the accuracy of the calculated ampacity. First, it is assumed that the properties of air are constant and may be evaluated at mid-range temperatures. This is reasonable because variations in heat capacity, conductivity, density and viscosity of air tend to compensate for one another and have very little net effect on heat transfer over the temperature range of interest. For example, the Prandtl number of air, $C\rho^{\mu/k}$ is commonly taken as 0.74 over a wide range of ordinary temperatures and pressures.

The properties used are as follows:

$C\rho$	heat capacity of air = 0.235 btu/lb.-°F
k	thermal conductivity of air = .018 btu/hr-ft^2-°F
$C\rho^{\mu/k}$	Prandtl number of air = 0.74
ρ^a	density of air = 0.062 lbs/cu Ft.
$\mu/\rho a$	kinematic viscosity = 0.9 ft^2/sec

As a result, only the temperature difference between the conductor and the surrounding air is important in calculating convective heat losses. For example, the convection losses calculated for a 40˚C temperature rise apply equally for a 70˚C conductor in 30˚C air or an 85˚C conductor in 45˚C air.

One might expect that the ampacities in the above instances would be different because the resistivities at 70˚C and 85˚C are different. However, it will be seen that the radiation losses which increase with the absolute temperature rather than the temperature difference tend to offset the rise in resistivities. As a result, ampacities based on the 40˚C ambient apply quite well to ambients from about 20˚C to 50˚C. Thus, for any temperature rise there is a single ampacity, (irrespective of the ambient) and it is usually not necessary to calculate a different ampacity for each ambient temperature and temperature rise.

C.3.2 Computation method

The general approach suggested for calculating the ampacity of any outdoor bus conductor is summarized below. A detailed explanation of each item follows.

Step by step the procedure is as follows:

1) Identify all exterior surfaces which should be treated as flat planes subject to forced convection.
2) Identify any exterior surfaces which should be treated as cylindrical surfaces subject to forced convection.
3) Identify any surfaces which may be shielded from the wind and only lose heat via natural convection (the same as indoors).
4) Identify surfaces which will lose heat also by radiation.
5) Ascertain the orientation and location of the conductors in determining the projected area exposed to solar heat gain.
6) For each of the appropriate areas (items 1, 2 and 3) compute the total convective heat losses, q_c.
7) For the appropriate values of emittance and area (item 4) compute the total heat lost through radiation, q_r .
8) Consider the projected area, latitude, altitude, seasonal factors, absorptivity, etc. and compute the solar heat gain, q_s.
9) Sum the heat gain and loss terms and, for the appropriately temperature compensated values of resistance *(R)* and skin effect coefficient *(F)*, compute ampacity using the general formula

$$I = \sqrt{\frac{q_c + q_r - q_s}{RF}}$$

where

I = current for the allowable temperature rise, amps.
q_c = convective heat loss, watts/ft.
q_r = radiation loss, watts/ft.
q_s = solar heat gain, watts/ft.
R = direct current resistance at the operating temperature, ohms/ft.
F = skin effect coefficient for 60 cycle current.

The following is an analysis of each of the individual operations. It will show that the basic equations can be reduced to easy to handle forms.

C.3.2 1. Forced convection over flat surfaces

When air flows parallel to and over a flat planar surface the following equation may be used to calculate the heat transfer coefficient:

$$h = 0.66(Lv\rho a/\mu)^{-1/2}(C\rho\mu/k)^{-2/3}(C\rho v\rho a)$$

where

h = heat transfer coefficient, btu/hr-°F ft^2
L = length of flow path over conductor (normally the width or thickness), in feet
v = air velocity, feet/hour

The total heat lost (in watts/ft) from the surface due to forced convection is

$$q_c = 0.00367 hA\Delta T$$

where

q_c = convection losses, watts/ft.
h = heat transfer coefficient BTU/hr ˚F ft^2
A = area of flat surfaces, square inches/linear foot
ΔT = temperature differences between the surface of the conductor and surrounding air, ˚C

At elevations above sea level multiply q_c by $P^{0.5}$ where P is the air pressure in atmospheres. This will reduce the convective coefficient for lower pressures.

For the properties of air noted earlier,

$$q_c = 0.0085 \sqrt{\frac{V}{l}} A\Delta T$$

where

v = air velocity, feet/sec.
l = length of surface over which air flows, inches (=12L)
For v = 2 feet per second

$$q_c = 0.012 \ A \sqrt{\frac{1}{l}} \Delta T$$

This simplified formula applies to air flow parallel to the surface. Outdoors air flow is seldom unidirectional and cannot always be parallel to the surface. However, it is assumed that air circulating around the conductor will be in more turbulent flow and provide on the average greater heat transfer than would be calculated using the above equation.

The convective loss formula above must be applied to each flat surface of the conductor. For example, consider a rectangular conductor 6″x 1/2″ operating at 100˚C in a 40˚C ambient. For the 6-inch faces A = 2 × 6 × 12 = 144 in^2/ft. Then

$$q_{c6} = \frac{(0.0120)(144)(60)}{6^{1/2}}$$

or

q_{c6} = 42.3 watts/ft.

For the 1/2-inch edges, A = 2 × (1/2) × 12 = 12 in^2/ft and $\sqrt{(1/2)}$ = .707.

Then

$q_c(1/2)$ = 12.2 watts/ft.

$q_c = q_c(1/2) + q_{c6}$ = 54.5 watts/ft.

Note that for a 6-inch square tube the convective heat loss would have been twice q_{c6} calculated above or 84 watts/ft. The heat loss per unit area, q_c/A, is 84/288 or 0.29 watts/in^2. It will be interesting to compare this value with that calculated for a 6-inch cylindrical pipe by a different method in the next section.

C.3.2 2. Forced convection over cylindrical surfaces

From McAdams [2][10] text or Perry's Handbook [1] heat transfer for a cylindrical shape at least 1-inch in diameter may be estimated as follows when there is a 2 fps wind and 1 atmosphere pressure

$$q_c = 0.010 \; d^{-0.4} A \Delta T$$

where

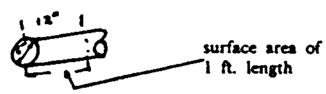
d = diameter of the cylinder, inches

A = Surface area in^2/ft

ΔT = Difference in temperature in ˚C between conductor surface and ambient air temperature

Thus, for a hypothetical pipe with an O.D. of six inches and $\Delta T = 60$ ˚C

$$A = 6 \times 3.14 \times 12 = 226 \text{ in.}^2/\text{ft}$$



$$q_c = (0.010)(6^{-0.4})(226)(60)$$

$$= \frac{(0.010)(226)(60)}{2.04}$$

$$= 66.8 \text{ watts/ft.}$$

The heat transfer per unit area is q_c/A or .298 watts/in^2. This value is virtually identical to that calculated for the square tube of the same major dimension and may be taken as an indication of the credibility of both methods.

It is of interest to make the comparison between square tubes and pipes for conductors of other size.

	q_c/A, watts/in^2	
Major Dimension (*d* or *l*) in inches	**Square Tube** $q_c/A = \frac{(0.0120)(60)}{l^{1/2}}$	**Pipe** $q_c/A = \frac{(0.010)(60)}{d^{-0.4}}$
3	0.415	0.386
6	0.293	0.293
9	0.240	0.248

[10]The numbers in brackets correspond to those of the references at the end of this annex.

It is seen that for the larger bus conductors the heat transfer efficiency of the pipe is about the same as that of the square tube. In fact they are identical at about 6 inches. Note that the heat transfer efficiency decreases with increasing size of the conductor.

C.3.2 3. Natural convection for flat and cylindrical surfaces

Some surfaces on conductors or in arrays of conductors may be shielded from direct exposure to wind. Assuming that there is nevertheless sufficient space for natural convection to occur, such surfaces may be treated as though convective losses outdoor would be the same as natural convective losses indoors. For such shielded surfaces heat losses are calculated using generally accepted equations for natural convection.

Examples of areas requiring such treatment are the spaces between double angles, double channels, or parallel rectangular conductors. The use of the natural convection equations is probably justified when the space between conductors is greater than 20% of the major dimension of the conductor or 1-inch, whichever is smaller. This estimate of the permissible spacing is based on the fact that the boundary layer for mass transfer is, very roughly, 10% of the length of the flow path. When the spacing between conductors is greater than the major dimension of the conductor, then the forced convection formulas given above may apply.

Because of the restricted flow away from the interior surfaces of integral web conductors, it is suggested that the natural convection loss formulas given here for surfaces facing down be applied to all interior surfaces.

The appropriate natural convection formulas are as follows:

Vertical or upward facing surfaces and cylinders

$$q_c = 0.0022\Delta T^{1.25} l^{-0.25} A$$

Surfaces facing down

$$q_c = 0.0011\Delta T^{1.25} l^{-0.25} A$$

where

ΔT = difference in temperature between conductor surface and ambient air temperature in ˚C
l = length of conductor surface (width or thickness) in inches (12L)
A = conductor surface area in inches^2/foot
q_c = conductive heat loss in watts/linear foot

For 3, 6, and 9 inch wide vertical surfaces at a 60˚C temperature difference

$$3'' \quad q_c/A = \frac{0.0022(60)^{1.25}}{3^{0.25}} = 0.28 \text{ watts/in}^2$$

$$6'' \quad q_c/A = 0.234 \text{ watts/in}^2$$

$$9'' \quad q_c/A = 0.21 \text{ watts/in}^2$$

When surfaces face downward the heat transfer per unit area is only half the value calculated in the above example.

Considering some other temperature differences, we get the following comparison between forced convection and natural convection.

Example

For a 6-inch flat conductor

q_c/A, watts/in^2			
	ΔT=80˚C	**ΔT=60˚C**	**ΔT=40˚C**
Forced convection (outdoor)	.390	.293	.195
Natural convection (indoor or confined spaces)	.335	.234	.141
Indoor/outdoor	.86	.795	.725

Thus, for large conductors and large temperature rises the calculated benefit of the 2 fps wind of heat transfer outdoors over natural convection on favorably oriented surfaces indoors is only 10-20%. The effect on ampacity will be even less and may be as low as only 2 or 3% for large conductors and high temperatures.

C.3.2 4. Radiation loss

The basic Stefan-Boltzmann equation for radiation from a surface (or narrow slits, which are treated as black bodies) is as follows:

$$q_r = 36.9 \times 10^{-12} \ \varepsilon \ A(T_c^{\ 4} - T_c^{\ 4})$$

where

- ε = emissivity corresponding to the temperatures of interest. Here is assumed emissivity at T_c equals absorptivity of energy spectrum at T_a. This is usually a good approximation.
- T_c = temperature of conductor, ˚Kelvin
- T_a = temperature of surrounding bodies, ˚Kelvin
- q_r = radiation loss watts/linear foot

Typical values of ε for bus conductors are in the range of 0.3 to 0.9. A value of 0.5 would apply to heavily weathered aluminum while 0.8–0.85 is appropriate for copper which has achieved a dense green or black-brown patina. High values of emittance may be achieved also with special paints, coatings or wrappings on the conductor. While high emittance improves heat dissipation via radiation it would also increase heat gain via solar absorption.

Example

Consider the conductor of emittance equal of 0.5 operating at 100˚C (373˚K) in an environment of 40˚C (313˚K) then

$$q_r/A = (36.9 \times 10^{-12})(0.5)(373^4 - 313^4)$$

$$q_r/A = .18 \text{ watts/in}^2$$

By comparing this figure to the forced convective losses calculated earlier it can be seen that radiation losses may make up 30–40% of the total heat losses. For large conductors with high emissivity, losses by radiation may exceed those due to convection.

C.3.2 5. Solar heat gain

The heat gained from incident solar radiation is estimated as follows:

$$q_s = 0.00695\varepsilon^6 Q_s A^9 K(\sin\theta)$$

where

ε^6 = coefficient of solar absorption, usually somewhat higher than emmitance, but generally taken as equal to that used for radiation loss
θ = effective angle of incidence of sun, $\cos^{-1}[\cos H_c \cos(Z_c - Z_1)]$
q_s = solar heat gain in watts/linear foot

where

H_c = altitude of sun, degrees
Z_c = azimuth of sun, degrees
Z_1 = azimuth of conductor line, degrees
= 0 or 180 for N-S
= 90 or 270 for E-W
A^9 = projected area of conductor, square inches per foot (area casting shadow)
Q_s = total solar and sky radiated heat on a surface normal to sun's rays, watts/sq.ft
K = heat multiplying factors for high altitudes

In cases where solar heat input is high, it is important to consider whether solar heating will peak during the time the maximum current load is on the circuit. If not, the estimate of the solar load should be reduced accordingly in order to arrive at the most cost-effective conductor size.

The projected area of a flat surface is the area of its shadow on a plane normal to the direction of the sun's rays, e.g., per foot of conductor.

$$A^9 = 12\sin\zeta \times \text{conductor size}$$

where
ζ = angle between plane of the conductor surface and sun's altitude

For a vertical surface
ζ = $90 - H_c$

For a horizontal surface
ζ = H_c

Table C.1—Data for calculating solar heat gain

Altitude and Azimuth in Degrees of the Sun at Various Latitudes[314] Declination 23.0° Northern Hemisphere • June 10 and July 3						
Degrees North Latitude	10:00A.M.		12:00 N.		2:00 P.M.	
	H_c	Z_c	H_c	Z_c	H_c	Z_c
20	62	78	87	0	62	282
25	62	88	88	180	62	272
30	62	98	83	180	62	262
35	61	107	78	180	61	253
40	60	115	73	180	60	245
45	57	122	68	180	57	238
50	54	128	63	180	54	232
60	47	137	53	180	47	223
70	40	143	43	180	40	217

$H_c = 62°$

Table C.1—Data for calculating solar heat gain (Continued)

Total Heat Received by a Surface at Sea Level Normal to the Sun's Rays[a]		
Qs watts/sq ft		
Solar Altitude Degrees HC	Clear Atmosphere	Industrial Atmosphere
5	21.7	12.6
10	40.2	22.3
15	54.2	30.5
20	64.4	39.2
25	71.5	46.6
30	77.0	53.0
35	81.5	57.5
40	84.8	61.5
45	87.4	64.5
50	90.0	67.5
60	92.9	71.6
70	95.0	75.2
80	95.8	77.4
90	96.4	78.9

[a]See Reference 5 at the end of this annex.

Table C.1—Data for calculating solar heat gain (Continued)

Solar Heat Multiplying Factors (K) for High Altitudes[a]	
Elevation above Sea Level, feet	**Multiplier for Q_s**
0	1.00
5,000	1.15
10,000	1.25
15,000	1.30

[a]See Reference 6 at the end of this annex.

Examples of Solar Heating
Example 1

Assume conductors are in an industrial area on a E-W line at 30˚N latitude at 5000 foot elevation. If maximum current is required, at 10:00 a.m. from Table C.13.

$Z_c = 98^\circ$
Q_s industrial = 72.3 watts/ft^2
K= 1.15 at 5,000 feet

Then,

$\theta = \cos^{-1} [\cos (62) \cos (98\text{-}270)]$

$\therefore \theta = 117.5^\circ$

$\sin \theta = 0.885$

For a cylinder, the projected area is 12d (in^2/ft). Then for a 6-inch cylinder with ε=0.8.

$$q_s = (0.00695)(0.8)(72.3)(12 \times 6)\ (0.885)(1.15)$$

$$q_s = 29.5 \text{ watts/ft.}$$

Example 2

Compute typical 10:00 a.m. summertime solar radiation incident on a 6x1/2-inch rectangular bus conductor running E-W at 45˚N latitude in a clear atmosphere at 5000 feet.

From Table C.1, $H_c = 57^\circ$

Projected area equals

$A' = 12 [6 \sin 33^\circ + 1/2 \sin 57^\circ]$
$A' = 44.28$ in^2/ft.
$\theta = \cos^{-1} [(\cos 57^\circ)[\cos(122^\circ - 270^\circ)]]$
$\theta = \cos^{-1} [(0.545)(0.53)] = \cos^{-1} (.\text{-}293)$
$\theta = 107^\circ$

$\sin\theta = .96$

$q_s = (0.00695)(0.5)(92)(44.28)\ (1.15)\ (.96)$

$q_s = 15.6$ watts/ft.

For comparison, consider the radiation loss for the same conductor at 80°C with 40° ambient.

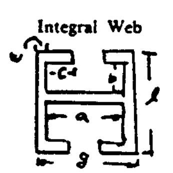
$$q_r = (36.9 \times 10^{-12})(0.5)(12)(12+1)(353^4 - 312^4)$$

$$= 17.0 \text{ watt/ft.}$$

This is a case where emissivity (absorptivity) is of minor importance in the rating of a bus conductor. In contrast, at a lower altitude and for a greater temperature rise, high emissivity would provide for improved ampacity. It should be noted that except during periods of peak solar loads, high emissivity provides the lowest operating temperatures and therefore the least power loss.

C.3.2 6. Summation of convective losses

For each of the conventional types of bus conductor, the convective loss areas for which the formulas given in items 1, 2, and 3 apply are as follows.

Shape	Area for Forced Convection	Area for Natural Convection	Summation of Convection Losses
Single Rectangle	24 (l+t)	0	$0.288\ \Delta\ T(l^{1/2} + t^{1/2})$
Multiple (N) Rectangles	24(l+Nt)	24l(N-1)	$0.288\ \Delta\ T(l^{1/2} + Nt^{1/2})$ + $0.0528\ \Delta\ T^{1.25}\ l^{.75}$ (N-1)
Round Tube or Bar	12πd	0	$0.377\ \Delta\ T\ d^{0.6}$
Square Tube	48l	0	$0.576\ \Delta\ Tl^{1/2}$
Rectangular Tube (l ×w)	24(l+w)	0	$0.288\ \Delta\ T(l^{1/2} + w^{1/2})$
Universal Angle (l ×w)(ignoring thickness)	24(l+w)	0	$0.288\ \Delta\ T(l^{1/2} + w^{1/2})$
Double Angles (for 2 angles)	24(l+w)	24(l+w)*	$0.288\ \Delta\ T(l^{1/2} + w^{1/2})$ + $0.0462\ \Delta\ T^{1.25}\ (l^{.75} + w^{.25})$
Single Channel	24(l+2w)	0	$0.288\ \Delta\ T(l^{1/2} + 2w^{1/2})$
Double Channel	24(l+2w)	24(l+2w)*	$0.288\ \Delta\ T(l^{1/2} + 2w^{1/2})$ $0.0462\ \Delta\ T^{1.25}\ (l^{.75} + 2w^{.75})$
Integral Web 	24(l+2w)	24(a + 2b + 2c)**	$0.288\ \Delta\ T(l^{1/2} + 2w^{1/2})$ $0.0264\ T^{1.25}\ (a^{.75} + 2b^{.75} + 2c^{.25})$

* Average over all surfaces on interior assuming equivalent of 3 favorably oriented surfaces and 1 unfavorable $\left[\frac{3(0.0022) + (0.0011)}{4}\right]24 = 0.0462$

** Due to overhang count all interior surfaces as unfavorably oriented for natural convection.

C.3.2 7. Summation of radiation losses

For each of the conventional bus conductors the areas which radiate energy are as follows

Shape	Surface Area of Material	Areas Which Behave as Black Body Slit or Hole ($\varepsilon'=1$)	Summation of Radiation Loss ÷ $(T_c^4-T_s^4) \times 10^{-12}$
Single Rectangle	$24(l + t)$	0	$886\varepsilon(l + t)$
Multiple (N) Rectangles (Spacing = S)	$24(l + Nt)$	(N-1)24S	$886\varepsilon(l + Nt + 886(N-1) S$
Round Tube or Bar Square Tube	$12\pi d$ $48l$	0 0	$1{,}390\varepsilon\, d$ $1{,}772\varepsilon\, l$
Rectangular Tube ($l \times w$)	$24(l + w)$	0	$886\varepsilon\, (l + w)$
Universal Angle	$24(l + w)$	0	$886\varepsilon\, (l + w)$
Double Angle (Two Angles) (Spacing = S)	$24(l + w)$	24S	$886\varepsilon\, (l + w + S/\varepsilon)$
Channel	$24(l + 2w)$	0	$886\varepsilon\, (l + 2w)$
Double Channel (Two Channels) (Spacing = S)	$24(l + 2w)$	0	$886\varepsilon\, (l + 2w + S/\varepsilon)$
Integral Web (overall dimensions $l \times g$)	$24(l + g)$	0	$886\varepsilon\, (l + g)$

C.3.2 8. Summation of solar radiation gains

The effective projected area for each of the conventional shapes is given below. Only direct solar radiation has been considered. A smaller amount of energy is radiated from the sky. However, it has been ignored here. If data is available for the particular ocation, sky radiation impinging on other surfaces may be added to the overall energy balance.

Shape	Effective Projected Area
Single Rectangle	$12[l \sin(90-H_c) + t \sin H_c]$
Multiple (N) Rectangles	$12[l \sin(90-H_c) + (Nt + (N-1)S/\varepsilon) \sin H_c]$
Round Tube or Bar	12d
Square Tube	$12l\, [\sin(90-H_c) + \sin H_c]$
Rectangular Tube ($l \times w$)	$12[l\sin(90-H_c) + w \sin H_c]$
Universal Angle	$12[l\sin(90-H_c) + w \sin H_c]$
Double Angle	$12[l\sin(90-H_c) + w \sin H_c]$
Channel	$12[l\sin(90-H_c) + w \sin H_c]$
Double Channel	$12[l\sin(90-H_c) + (2w + S/\varepsilon) \sin H_c]$
Integral Web	$12[l\sin(90-H_c) + 2w \sin H_c + [(g-2w)/\varepsilon)] \sin H_c]$

C.3.2 9. Computation of ampacity

The ampacity computation requires dividing the sum of the heat losses by the product of the resistance (*R*) and the skin effect factor (*F*).

Resistance increases with increasing temperature and this must be accounted for in the calculation. Skin effect factors are a function of resistance, frequency and geometry. The factors are readily available for simple shapes. Calculating skin effect factors for complex shapes is beyond the scope of this paper and no guidance will be offered except that the factors can be significant and should be included when calculations are performed. The skin effect factors decrease slightly with increasing temperature and should be adjusted accordingly. This subject is discussed in the section on properties of materials.

As shown in the section on properties of materials, the resistance at any temperature may be calculated as follows:

For copper and copper alloys

$$R = \frac{8.145 \times 10^{-4}}{C' A_2}\left[1 + \frac{0.00393C'}{100}(T_2 - 20)\right]$$

For aluminum alloys

$$R = \frac{8.145 \times 10^{-4}}{C' A_2}\left[1 + \frac{0.00403C}{61}(T_2 - 20)\right]$$

where

C' = conductivity as % IACS
A_2 = cross-sectional area, square inches
T_2 = conductor temperature, °C

Example

Compute the 60 cycle outdoor ampacity of a 12″ by 1/4″ copper conductor operating with a temperature rise of 65°C above a 40°C ambient. Assume ε=0.5, no solar heating, C'=98% IACS and F=1.28

$$q_c = (0.288)(65)[12^{1/2} + (1/4)^{1/2}]$$

$$q_c = 74 \text{ watts/ft}$$

$$q_r = (886)(0.5)(12.250)(10^{-12})(378^4 - 313^4)$$

$$q_r = 58.2 \text{ watts/ft}$$

$$q_c + q_r = 132.2 \text{ watts/ft}$$

$$R = \frac{8.145 \times 10^{-4}}{(98)(12)(1/4)}\left[1 + \frac{0.00393(98)}{100}(105 - 20)\right]$$

$$= 3.68 \times 10^{-4} \text{ ohms/ft}$$

$$RF = (3.68)(1.28) \times 10^{-4} = 4.7 \times 10^{-4} \text{ ohms/ft}$$

$$I = [(q_c + q_r)/RF]^{1/2} = 10^3 \ (132.2/4.7)^{1/2}$$

$$I = 5,310 \text{ amps}$$

C.4 Properties of materials

C.4.1 Thermal expansion

Bus conductors expand as their temperatures rise. The amount of expansion may be calculated by multiplying the coefficients below by the increase in temperature. The base temperature corresponding to zero expansion is the installation temperature not the ambient temperature.

Material

Table C.2—Thermal expansion multiplication coefficients

	Average Coefficient of Thermal Expansion for the Range Indicated	
	in/in-˚F (68-212˚F)	**in/in˚C (20-100˚C)**
Aluminum and Alloys	13.0×10^{-6}	23.4×10^{-6}
Copper and Alloys	9.22×10^{-6}	16.6×10^{-6}
Steel	6.3×10^{-6}	11.4×10^{-6}
Concrete	3.5 to 8×10^{-6}	6.3 to 14.4×10^{-6}

Example

What is the total thermal expansion of a 15-foot run of copper bus conductor installed on a concrete pad at 20˚C and operating at 50˚C over a 40˚C ambient (i.e. at 90˚C)

For the bus conductor
Δ copper = total expansion = $(12)(15)(16.8 \times 10^{-6})(70) = 0.211$ inches

For the concrete pad (assume coefficient expansion = 10×10^{-6})
Δ concrete = $(12)(15)(10 \times 10^{-6})(20) = 0.036$ inches

Net amount of restraint on bus conductor is the difference between the expansion of the bus and the concrete pad

Δ net = 0.211 – 0.036 = 0.175 inches

The strain on the copper (assuming massive rigid pad) is

$$\frac{\Delta net}{L'} = \frac{0.175}{12 \times 15} = 0.001 \text{ inches/inch}$$

where

L' = length of restrained conductor in same units as Δ net

C.4.2 Stresses and forces due to thermal expansion

When a material is totally restrained from expanding or contracting normally as temperatures change, stresses are induced to account for the effective change in length.

The stress, S, is

$$S = \frac{E\Delta net}{L'}$$

where

E = modulus of elasticity

For the materials of construction

Table C.3—Modulus of elasticity

	E, modulus of elasticity, $\times 10^4$ psi		
	20˚C	**50˚C**	**100˚C**
Aluminum	10	10	10
Copper	17	16.5	16
Steel	30	30	30
Concrete	3 to 5	3 to 5	3 to 5

Example

For the example above

$S = 17 \times 10^6 \times 10^{-3} = 17{,}000$ psi

The total load is $S \times A_2$

where

A_2 = cross-sectional area, sq. inches

For 6″×1/2″ bus conductor the associated load on the bus supports in the above case would be 51,000 pounds. In practice this high load would not be generated. Complete restraint is unlikely due to bending, sliding, or plastic deformation of the conductors. However, to be sure loads are not excessive it is suggested that expansion joints be provided to minimize thermally generated stresses.

C.4.3 Maximum operating stresses

Metals may deform plastically to accommodate thermal stresses and strains and reduce other applied loads. While bus conductor alloys can deform appreciably it is suggested that stresses be maintained below levels at which plastic deformation is expected. If the loads will be applied occasionally and for only a short time the maximum stress should be below the yield strength. It must be remembered that, at the yield stress of a material, a small amount of deformation (less than 1/2 percent) occurs. For extended operation or negligible deformation lower stresses must be employed to avoid creep, relaxation or fatigue damage. To provide a margin of safety designers may limit stresses to 2/3 the values given below in Table C.4.

Table C.4—Operating stresses

Representative Yield Strength Levels, psi			
	20˚C	**100˚C**	**150˚C**
Aluminum Alloys			
6101-T6	25 000	22 300	16 900
6063-T6	25 000	22 700	16 200
Copper (Hard)	25 000	22 000	20 000
Copper (Soft)	9 000	9 000	9 000
Maximum Stresses for Continuous Operation, psi			
	20˚C	**100˚C**	**150˚C**
Aluminum Alloys			
6101-T6	15 000	13 380	10 140
6063-T6	15 000	13 620	9 720
Copper (Hard)	9 000	9 000	8 700
Copper (Soft)	5 100	4 800	4 700

The above strength levels apply to the usual conductor materials. Special alloys of aluminum or copper and coppers with small additions of silver may be used where higher strength or resistance to relaxation or softening are required.

C.4.4 Resistance

Resistance of bus conductors increases with increasing temperature. For aluminum and copper alloys, resistance at an elevated temperature (T_2) may be expressed in terms of resistance at 20˚C as follows

$$R_{T2} = R_{20}[1 + \alpha(T_2 - 20)]$$

where

$\propto$ = temperature coefficient of resistance for a base of 20°C (ohms/ohms-°C)

R_{20} = resistance at 20°C per unit length in ohms/foot
= ρA_2

where

ρ = resistivity, ohm-in³/ft
A_2 = cross-sectional area of conductor at 20°C, in sq. in.

The temperature coefficient of resistance for copper of conductivity equal to 100% of the International Annealed Copper Standard (IACS) is 0.00393/°C and for aluminum of conductivity equal to 61% IACS it is 0.00403/°C. For copper and aluminum conductors of other conductivities the following relations may be written for

C´ = % conductivity (as % IACS)

$$\alpha \ cu = \frac{0.00393C'}{100}$$

$$\alpha \ \text{al} = \frac{0.00403C'}{61}$$

The above relations give the following for copper

$$R_{\text{T2}} = \rho / A_2\left[1 + \frac{0.00393C'}{100}(T_2 - 20)\right]$$

and for aluminum

$$R_{\text{T2}} = \rho / A_2\left[1 + \frac{0.00403C'}{61}(T_2 - 20)\right]$$

For copper of 100% IACS conductivity the resistivity, ρ, is 8.145×10^{-6} ohm-in^2/ft. Then

Copper

$$R_{\text{T2}} = \frac{8.145\times10^{-6}}{C'A_2}\left[1 + \frac{0.00393C'}{61}(T_2 - 20)\right]$$

Aluminum

$$R_{\text{T2}} = \frac{8.145\times10^{-6}}{C'A_2}\left[1 + \frac{0.00403C'}{100}(T_2 - 20)\right]$$

C.4.5 Emissivity and absorptivity

For ordinary calculations the emissivity and absorptivity of a bus conductor are taken as equal. Strictly speaking, since they apply to different energy spectra they are not equal, but for practical purposes the error is small.

For conditions of interest,

Table C.5—Emissivity and absorptivity of material

ε = Emissivity, absorptivity		
	Copper	Aluminum
Clean Mill Finish	0.1	0.1
Light Tarnish (recent outdoor installation or indoor)	0.3–0.4	0.2
After Extended Outdoor Exposure	0.7–0.85	0.3–0.5
Painted Black	0.9–0.95	0.9–0.95

C.4.6 Skin effect

For common conductor shapes plots are available which provide skin effect coefficients as a function of current frequency and resistivity. When such plots are available the variation in skin effect with temperature may be determined by computing the resistivity of the shape at various temperatures and determining the associated skin effect coefficients. When only a single value of the skin effect coefficient is suitable or when a convenient equation is needed for computer calculations, the following procedure may be used to obtain a conservative (slightly) high estimate of the skin effect coefficient at a higher temperature.

For

F_1 = skin effect coefficient at temperature T_1
F_2 = skin effect coefficient at temperature T_2

Then

$$F_2 = F_1 + \frac{\Delta F}{\Delta T}(T_2 - T_1)$$

Normally the skin effect coefficient is given as a function of Frequency/Resistivity × 10^3 which we will define as X for convenience here. Then

$$\frac{\Delta F}{\Delta T} \approx \frac{dF}{dt} = \frac{dF}{dX}\frac{dX}{dR}\frac{dR}{dt}$$

$$\frac{dX}{dR} = -\frac{X}{2R}$$

$$\frac{dR}{dt} \approx R\alpha$$

A conservative estimate of dF/dX is always $(F\text{-}1)/X$. Then

$$\frac{\Delta F}{\Delta T} \approx -\frac{(F-1)}{X} \times \frac{1}{R} \times \frac{X}{R} \times R\alpha$$

$$\approx -1/2\alpha(F-1)$$

Therefore

$$F_2 = F_1 - \frac{1}{2}(T_2 - T_1)(F_1 - 1)\alpha$$

C.5 Ampacity tabulations

The procedures described herein have been used to calculate ampacity tables which are a separate document.

C.6 References

[1] *Chemical Engineer's Handbook*, J. H. Perry, ed. McGraw-Hill Book Company, 1950. Chapter 6 by McAdams, W. H.

[2] McAdams, W. H., *Heat Transmission,* McGraw-Hill Book Co., N.Y., 1954.

[3] *The American Nautical Almanac,* U.S. Naval Observatory, Washington, D.C., 1957.

[4] *Sight Reduction Tables for Air Navigation,* U. S. Navy Hydrographic Office, H. O. Publication No. 249, Vols. II and III.

[5] *Heating, Ventilating and Air-Conditioning Guide 1956,* American Society of Heating and Air-Conditioning Engineers.

[6] Yellot, J. I., "Power from Solar Energy," *ASME Transactions* Vol. 79, No. 6, AUgust, 1957, pp. 1349-1357.

Note 1

The wind is considered a forced draft with the air circulating parallel to each surface of the conductor and perpendicular to the length

This paper is part of the work of a task force of the IEEE Substations Committee's Working Group 69.1 "Rigid Bus Design Criteria for Outdoor Substations." Messrs. Bleshman, Pemberton, Craig and Prager are members of that task force.

Discussion

W. H. Dainwood, J. E. Holladay, and S. W. Kercel (Tennessee Valley Authority, Knoxville, TN: The authors should be commended on this paper in which they have presented a very sophisticated method of calculating the temperature rise for a certain value of current. It should become an important reference for design of rigid bus systems.

We are utilizing a procedure for calculating temperature rise that is similar to the authors' approach. However, at the present our computerized procedure is limited to tubular and solid round conductors. We use the equations for heat loss which are in the *Westinghouse Electrical Transmission and Distribution Reference Book*, copyright 1964, Fourth Edition, Fifth Printing. Also used as a reference is the book *Elements of Power System Analysis*, second edition, by William D. Stevenson, Jr. As with the equations in this paper, the ones we use express current as a function of temperature rise. Primarily, we are interested in specifying a value of current and determining the temperature rise. To do this, we use the Newton-Raphson technique to solve the equation which expresses the current as a function of temperature rise. Have the authors considered this approach?

We would suggest that the authors include, under "PROPERTIES OF MATERIALS" No. 6, *Skin Effect*, the method for calculating the skin effect ratio defined as $\frac{\text{AC resistance}}{\text{DC resistance}}$. It appears the authors have given a conservative method for estimating the skin effect ratio. This estimate approach seems to be somewhat in disagreement with the statement in the *ABSTRACT* which says, "This paper will allow the engineer to reexamine the factors involved in increased current loadings of rigid bus and possibly determine new thermal limits." If the object of the paper is to move away from conservative estimates and look at what is actually happening, then it appears that more explicit equations for skin effect could also be presented. We feel that this would further enhance a very significant paper.

The following is an extract from the computer program which we have developed:

Calculation of skin effect ratio:

The literature defines a quantity in

$$m = \sqrt{\frac{4\pi\omega}{\rho}}$$

where

ω = 2πf
ρ = DC resistivity in μohm-m

Stevenson demonstrates (*Power System Analysis*, pages 81-82):

$$mr = .0636 \sqrt{\frac{F}{Ro}}$$

where

F = 60 Hz
Ro = ohm/mil (DC)

or

$$mr = .0636 \sqrt{\frac{F}{Ro \ 5280}}$$

Ro ohm/ft (DC)

For a solid round conductor of radius *r*

Now it follows from this that:

$$m = \frac{mr}{r} = \frac{.0636}{r} \sqrt{\frac{F}{Ro \ 5280}}$$

Where Ro is the DC resistance in Ω/ft of a solid conductor of radius *r.*

By calculating m by this formula, you can be sure the units will come out right.

The ratio is (*Electrical Coils and Conductors*, page 172):

$$\frac{R_{AC}}{R_{DC}} = Re\left[\frac{\alpha(q-r)(q+r)}{2r}\left(\frac{I_o(\alpha r)K_{o'}(\alpha q) - K_o(\alpha qr)I_o(\alpha q)}{I_{o'}(\alpha r)K_{o'}(\alpha q) - K_{o'}(\alpha r)I_{o'}(\alpha q)}\right)\right]$$

where

$\alpha = \sqrt{j}\ m$

$j = \sqrt{-1}$

$I_o(\alpha r)$ = ber mr + j bei mr

$I_o(\alpha q)$ = ber mq + j bei mq

$K_o(\alpha r)$ = ker mr + j kei mr

$K_o(\alpha q)$ = ker mq + j kei mq

$I_o'(\alpha r) = e^{-j\pi/4}$ (ber′ mr + j bei′ mr)

$I_o'(\alpha q) = e^{-j\pi/4}$ (ber′ mq + j bei′ mq)

$K_o'(\alpha r) = e^{-j\pi/4}$ (ker′ mr + j kei′ mr)

$K_o'(\alpha q) = e^{-j\pi/4}$ (ker′ mq + j kei′ mq)

Where the following bessel functions are defined by infinite series:

$$\text{ber } ax = 1 - \frac{(ax/2)^4}{(2!)^2} + \frac{(ax/2)^8}{(4!)^2} - \ldots$$

$$\text{bei } ax = \frac{(ax/2)^2}{(1!)^2} - \frac{(ax/2)^6}{(3!)^2} + \frac{(ax/2)^{10}}{(5!)^2} - \ldots$$

$$\text{ber}' \ ax = -\frac{2a(ax/2)^3}{(2!)^2} + \frac{4a(ax/2)^7}{(4!)^2} - \ldots$$

$$\text{bei}' \ ax = \frac{a(ax/2)}{(1!)^2} - \frac{3a(ax/2)^5}{(3!)^2} + \ldots$$

$$\text{ker } ax = -\left(\ln\frac{ax}{2} + C\right) \text{ ber } ax + \frac{\pi}{4} \text{ bei } ax - \lambda 2 + \lambda 4 - \ldots$$

$$\text{kei } ax = -\left(\ln\frac{ax}{2} + C\right) \text{ bei } ax - \frac{\pi}{4} \text{ ber } ax + \lambda 1 - \lambda 3 + \ldots$$

where C = .57721 56649 0 1532 86061

$$\lambda K = \left(\frac{(ax/2)^{2k}}{(K!)^2}\right)\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots\frac{1}{K}\right)$$

$$\ker' ax = -\left(\ln\frac{ax}{2} + C\right)\, \mathrm{ber}' ax - \frac{1}{x}\mathrm{ber}\, ax + \frac{\pi}{4}\,\mathrm{bei}' ax - \lambda 2' + \lambda 4' - \ldots$$

$$\mathrm{kei}\, ax = -\left(\ln\frac{ax}{2} + C\right)\, \mathrm{bei}' ax - \frac{1}{x}\mathrm{bei}\, ax - \frac{\pi}{4}\ \mathrm{ber}' ax + \lambda 1' = \lambda 3' + \ldots$$

$$\lambda' K = \frac{2K}{x}\ \lambda K$$

M. Prager, D. L. Pemberton, A. G. Craig, and N. A. Bleshman: The authors thank Messrs. Dainwood, Holladay, and Kercel for their timely comments. The formulas presented in the paper were selected as the first stage in a program to develop ampacity tables for commercial bus conductors. Such tables may be used alternatively to determine the allowable current for a specified temperature rise or the temperature rise for a specified current. Many of these tables have been prepared based on these formulas and they will be the subject of a forthcoming paper.

When a quick estimate of the temperature rise for 2 given current is needed the following procedure may be used without the need for a computer. The temperature term in the expression for radiation loss (i.e., $T_2^4 - T_1^4$) may be approximated by 1.6 x $10^8\Delta T$ and the exponential term in the natural convection equations ($\Delta T^{1.25}$) may be approximated by 2.8ΔT. Substituting these terms into the general expression relating current to heat loss and resistance $I = \sqrt{\frac{qc + qr - qs}{RF}}$ provides an equation in which the unknown (ΔT) appears to the first power. The solution is then easily obtained by solving that equation.

In using the expression suggested by Messrs. Dainwood et. al. to calculate the skin effect ratio, it must be remembered that the temperature coefficient should be included in the resistivity term. The authors took the approach that since the skin effect ratios for conductors were usually available at one temperature, such data could be modified conveniently by the method shown in the text $F_2 = F_1 - 1/2(T_2 - T_1)(F_1 - 1)\ \alpha$ The error introduced is negligible for practical purposes.

Annex D

(informative)

Calculation of surface voltage gradient

The allowable surface voltage gradient E_o for equal radio-influence (RI) generation for smooth, circular conductors is a function of bus-conductor diameter, barometric pressure, and operating temperature. It may be calculated as follows:

$$E_o = \delta g_o \tag{D1}$$

where

E_o = allowable surface voltage gradient, kV rms/cm
g_o = allowable surface voltage gradient under standard conditions for equal radio-influence generation and for circular conductors, kV rms/cm (see Figure D.1)

$$\delta = \frac{7.05b}{459 + F}$$

where

δ = air density factor
b = barometric pressure, cm of H_g
F = temperature, °F

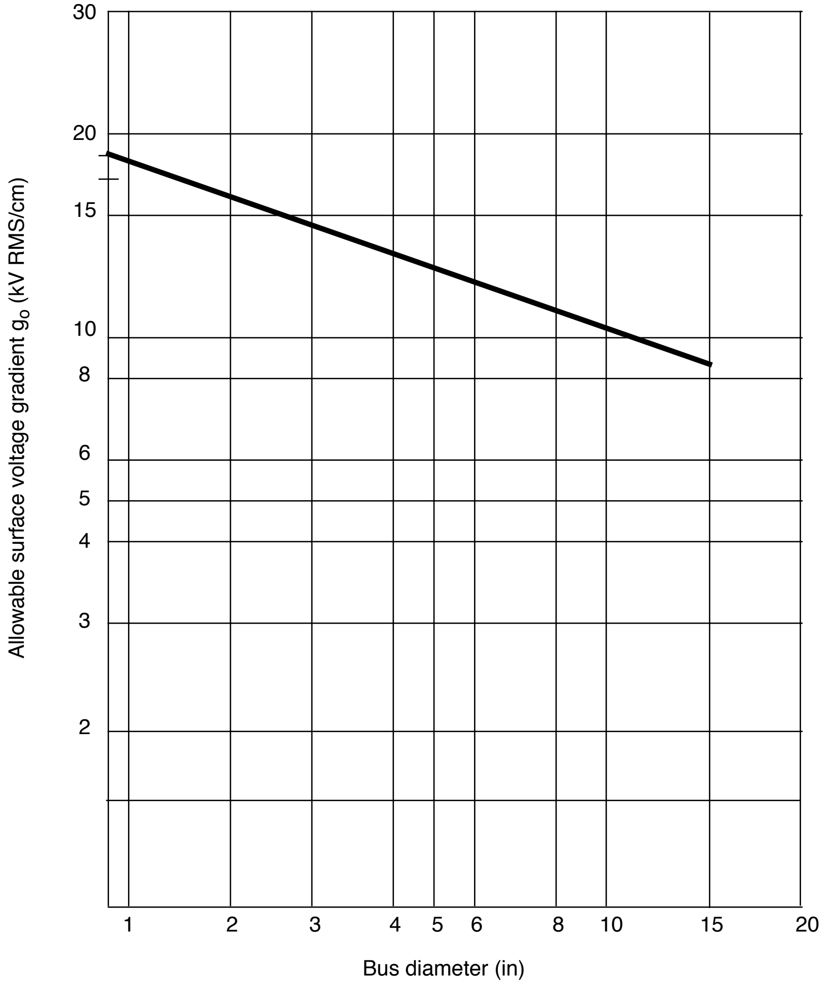


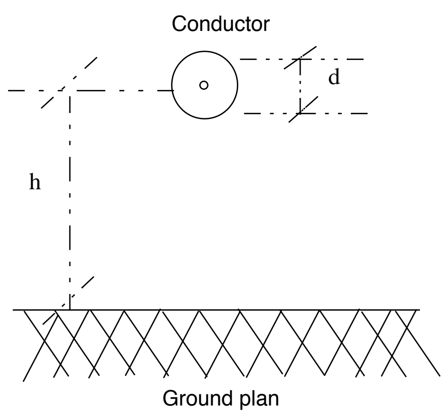
Figure D.1—Allowable surface voltage gradient for equal radio-influence generation under standard conditions versus bus diameter

The temperature to be used in Equation (D1) is generally considered to be the conductor operating temperature. Table D.1 gives standard barometric pressure corrected for various altitudes above sea level.

Table D.1—Standard barometric pressure (for various altitudes)

Altitude (ft)	Altitude (m)	Pressure (cm of H_g)
–1000	–300	79.79
–500	–150	77.39
0	0	76.00
1000	300	73.30
2000	600	70.66
3000	900	68.10
4000	1201	65.63
5000	1501	63.22
6000	1801	60.91
8000	2402	56.44
10 000	3003	52.27
15 000	4504	42.88
20 000	6006	34.93

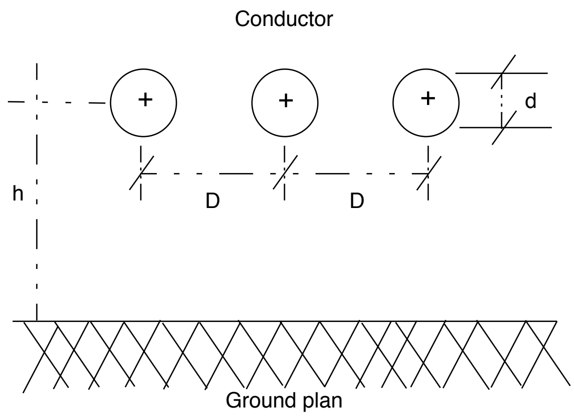
The average and maximum surface voltage gradients at the surface of smooth circular conductors, at operating voltage, may be determined by the following formulae from NEMA CC 1-1993.



$$E_a = \frac{V_1}{\frac{d}{2} L_n\left(\frac{4h}{d}\right)}$$

$$E_m = \frac{h}{h - \frac{d}{2}} E_a$$

Figure D.2—For single conductor



$$E_a = \frac{V_1}{\frac{d}{2}L_n\left(\frac{4h_e}{d}\right)}$$

$$E_m = \frac{h_e}{h_e - \frac{d}{2}}E_a$$

$$h_e = \frac{hD}{\sqrt{4h^2 + D^2}}$$

Figure D.3—For three-phase conductor

where

h = distance from center of conductor to ground plane, cm [in]
h_e = equivalent distance from center of conductor to ground plane for three phase, cm [in]
d = diameter of the individual conductor, cm [in]
D = phase-to-phase spacing for three phase, cm [in]
V_1 = line-to-ground test voltage, kV
E_a = average voltage gradient at the surface of the conductor, kV/cm [kV/in]
E_m = maximum voltage gradient at the surface of the conductor, kV/cm [kV/in]

NOTE—V_1 = 110% of nominal operating line-to-ground voltage

For the three-phase configuration the center conductor has a gradient approximately 5% higher than the outside conductors. For bundled circular conductors, formulae for calculating the surface voltage gradient may be obtained from NEMA CC 1-1993.

For satisfactory operation, E_m must be less an E_o.

Annex E

(informative)

Mechanical forces on current-carrying conductors

By E. D. Charles[11]

Synopsis

Following a brief review of the standard formulae in connection with the forces on current-carrying conductors the author examines the problem from a more general standpoint, and derives formulae for both the distribution and direction of forces on conductors lying at any angle in different planes. It is felt that these formulae [Equations (E4) and (E5)], which have not previously been published, will be of value for the following reasons: (i) The approximations obtained by application of the standard formulae to non-standard conductor arrangements may lead to serious over- or under-estimation of the true magnitude of mechanical forces and their moments. (ii) A precise knowledge of the direction of the resultant mechanical force is of considerable importance in determining the cantilever stress in the very long insulator stacks used for h.v. installations. (iii) The general formulae put forward are comprehensive in that all the standard formulae for the distribution and direction of forces may be readily obtained by suitable substitution.

List of symbols

d = shortest distance between centre-lines of two straight cylindrical conductors crossing each other obliquely in different planes, m

dF = mechanical force on element dx of conductor, N

$F_p\left(=\dfrac{dF}{dx}\right)$ = mechanical force per unit length at point P on conductor, N/m

F_h, F_v = horizontal and vertical components of F_p, N/m

$I_1 I_{2x}$ = current in conductors, A

χ = angle between direction of mechanical force on an element of conductor and the plane in which the conductor lies

α = angle between conductor and the direction of the magnetic field in which it lies

β = angle between one conductor and the trace of the other in a plane perpendicular to the shortest distance between the two conductors

E.1 Introduction

A large number of papers have been written in connection with the forces of attraction and repulsion between current-carrying conductors. Following the work of Ampère, Laplace, Biot, and Savart, the underlying principles were well established, and a number of other investigators formulated methods of computing the forces in several practical arrangements of conductors lying in a plane or crossing each other at right angles.

In the paper a general formula is given from which may be calculated the distribution of mechanical forces along current carrying conductors which lie at any angle in different planes.

[11]Published by Proceedings IEEE, Volume 110, No. 9, Sept. 1963.

E.2 Conductor arrangements

It is well known that adjacent current-carrying conductors experience a mechanical force which depends upon the magnitude of the current and the geometrical configuration of the conductors.

The forces which arise under short-circuit conditions may amount to several tons and must be taken into account in the design of conductors, insulators and their supporting structures. The calculation of the forces is a simple matter in the case of very long, straight, parallel busbars, because for all practical purposes the forces are uniformly distributed along the length of the conductors. At the extreme ends of the conductors the forces actually 'tail off' owing to the reduction of magnetic-field strength, but this so-called 'end effect' is only of importance in conductor arrangements in which short lengths, bends, taps, and cross-overs form part of the complete circuit. (Frick, [1])[12]

A knowledge of the way in which the mechanical forces are distributed along a conductor is a first requirement in computing both the total force and the moment of these forces about a particular point.

The total force on a section of conductor is obtained by integrating the force per unit length over the section. In a similar manner, the moment of the force on a particular section of conductor about a specified point is found by integrating the product of force per unit length times distance to the point. The mathematical integration of the expression for force per unit length is possible only in simple arrangements such as those shown in Figure E.1, so that, in the general case, graphical methods of investigation must be adopted.

The mechanical force on a particular conductor forming part of a complete circuit is found by summing the component forces calculated for the individual conductor members making up the circuit. The conductor members are treated in pairs, each member being taken in combination with every other member, although it is often possible to neglect the more remote parts of the circuit when it is estimated that their effects are negligible compared with other component forces.

It is assumed that the conductors are of circular cross section and that the current is concentrated along the axis of the conductor. No error is introduced by this latter assumption, since, neglecting proximity effects with alternating current, the external magnetic field due to current in a cylindrical conductor does not depend upon the radius of the conductor. Proximity effects need not be considered where the clearance between two members is more than twice the diameter of the conductor.

When the conductors are near together, the mechanical forces in conductors of rectangular cross-section are different from those in conductors of circular cross-section, and for further information the reader should consult the references [2] through [5].

Methods of calculating electromagnetic forces are presented in textbooks and papers for the following cases, illustrated in Figures E.1a, E.1b and E.1c:

a) Parallel conductors
b) Right-angled cross-over conductors
c) Conductors at any angle lying in a plane

The formula for case c) was first introduced by Dunton in 1927. (Dunton, [E6])

For ease of calculation, a complete circuit is usually simplified by regarding it as a combination of the arrangement *a)*, *b)* and *c)*, and a further simplification is often obtained in arrangement *c)* by assuming that the angle between the conductors is a right angle. Although these approximations suffice in many practical

[12]The numbers in brackets correspond to those of the references at the end of this annex.

cases, problems may arise where greater accuracy is desired, and in these circumstances a more detailed calculation may be justified.

E.3 Skewed-conductor arrangements

It will be realized that *a)*, *b)*, and *c)* are special cases of a more general arrangement in which the axes of the conductors are two straight lines skewed in space at any angle relative to each other, as in Figure E.2.

The definition of two skew lines is that they neither intersect nor are parallel, although *a* and *c* of Figure E.1 may be regarded as limiting cases.

In pure geometry it is shown that, if two lines JD and HA neither intersect nor are parallel, then (see Figure E.2)

(i) there is one straight line CB which is perpendicular to both the given lines
(ii) the length, *d*, of the common perpendicular is the shortest distance between the lines

It follows that JD and HA lie in two parallel planes separated by the distance CB. Thus the general case can be analyzed by using one conductor HA and the shortest distance CB to form the framework of reference shown in Figure E.3. The special cases shown in Figure E.1 are obviously obtained from Figure E.3 as follows:

a) $\beta = 0^\circ$ (parallel)
b) $\beta = 90^\circ$ (right-angled cross-over)
c) $d = 0$ (any angle in a plane)

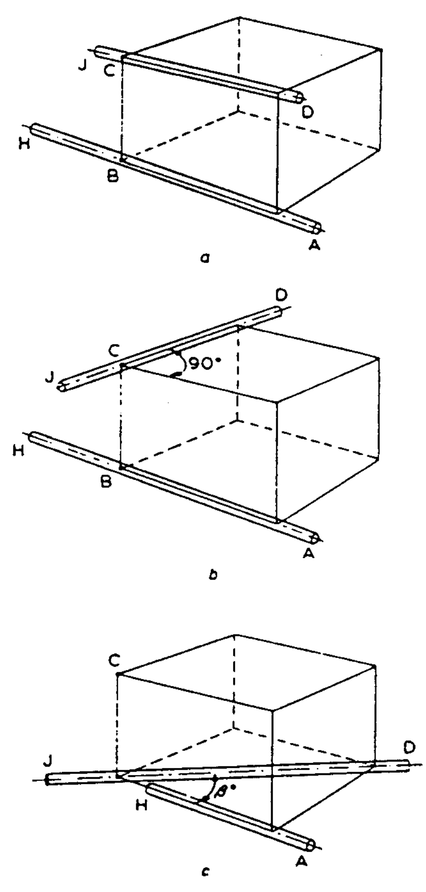


Figure E.1—Conductor arrangements—special cases

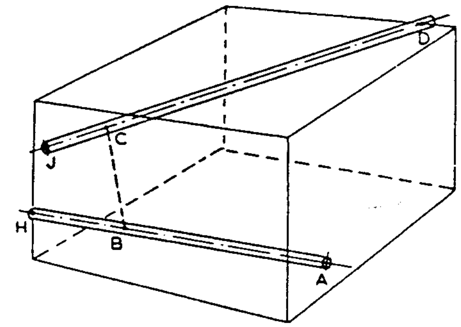


Figure E.2—Skewed conductors

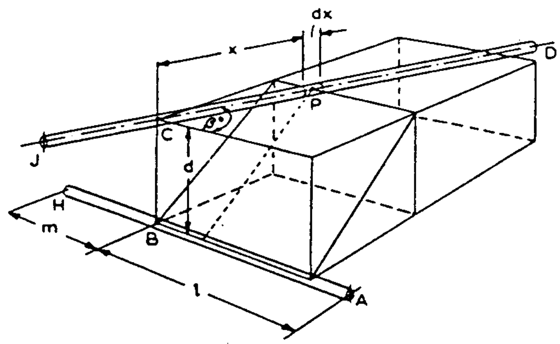


Figure E.3—Skewed conductors—reference axes and dimensions

E.4 Distribution and direction of forces

It has already been pointed out that the mechanical forces experienced by current-carrying conductors are not uniformly distributed along their length, the degree of non-uniformity being more pronounced in the case of short lengths, bends and cross-overs. The direction of the forces depends upon the relative directions of the currents. In the parallel arrangement, the conductors are attracted when the currents are in the same direction and repelled when the currents are in opposite directions. Two circuits crossing obliquely attract each other when both the currents proceed from or to the apparent point of intersection but repel each other if one current proceeds from and the other towards that point.

Figure E.4 shows the approximate distribution and the direction of forces in the three special cases *(a)*, *(b),* and *(c)* when the currents are flowing in directions such as to cause repulsion between the two conductors. If one of the currents is reversed, the direction of the forces will also be reversed. All forces are at right angles to the conductor. In these special cases it will be observed that the mechanical forces are uniplanar.

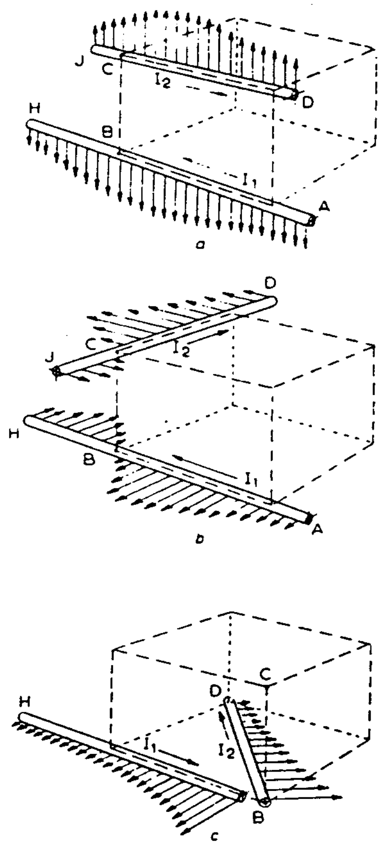
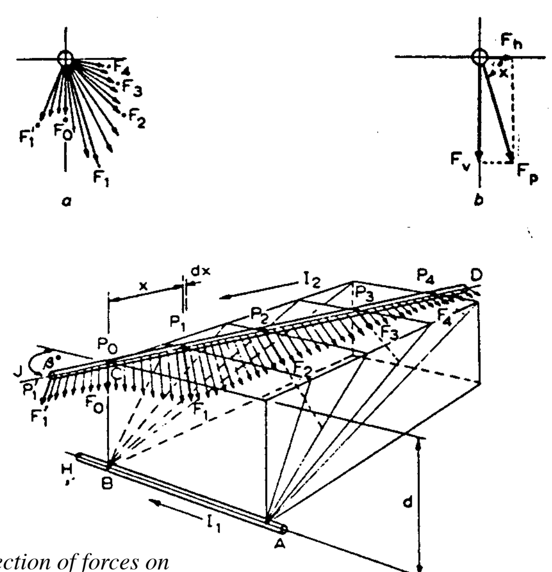


Figure E.4—Direction of forces—special cases



a *Magnitude and direction of forces on conductor JD viewed axially*

b *Orthogonal components of* F_p

Figure E.5—Direction of forces—skewed conductors

Figure E.5 shows the skewed-conductor arrangement carrying currents I_1 and I_2 in the directions shown. Consider an elemental portion *dx* of the conductor JD at point P_1. The direction of the magnetic flux at point P_1 due to the current I_1 in conductor HA is normal to the plane HP_1A. The mechanical force F_1 experienced by the element *dx* is in a direction at right angles to both the flux at point P_1 and to the conductor JD. The line from P_1 representing the force F_1 therefore lies in the plane HP_1A and is radial to the conductor JD. In the same way, the forces experienced by an element of conductor JD at any other point such as P_0, P_2, P_3, etc., are radial to JD and lie in the planes containing both the conductor HA and the point considered. The angles at which the forces F_0, F_1, F_2, etc., act depend upon the values of *x*, *d* and β. Figure E.5a shows the magnitude and direction of the forces as viewed along the axis of the conductor. Figure E.5b shows the horizontal and vertical components of F_p.

$$F_h = F_p \cos X$$

$$F_v = F_p \sin X$$

where, as shown in E.10.2, $X = \tan^{-1}(d/x \tan\beta)$.

E.5 Development of general formula for the distribution of mechanical forces in current-carrying conductors

E.5.1 Magnitude of force per unit length

In Figure E.6, HA and JD are two conductors of circular cross-section carrying currents I_1 and I_2 amperes, respectively.

Consider the force dF_s on a length *dx* of conductor JD at P due to current flowing in element *ds* of conductor HA at G. According to Ampère's law, the force between the current elements *dx* and *ds* is

$$\frac{I_1 I_2 dx ds}{z^2} \sin\phi \sin\alpha \times 10^{-7} \qquad \text{(E1)}$$

where

z = length of the line PG
ϕ = angle between conductor HA and the line z
α = angle between the normal to the plane HPA at
= P and the conductor JD.

Now GE = $ds \sin\phi$

Also GE = $zd\phi$

$$\sin\phi = \frac{zd\phi}{ds}$$

but $\frac{'k}{z} = \sin\phi$, so that $\frac{ds}{z^2} = \frac{d\phi}{k}$

Substituting in Equation E1 we get

$$\frac{I_1 I_2 dx d\phi}{k} \sin\phi \sin\alpha \times 10^{-7} \qquad \text{(E2)}$$

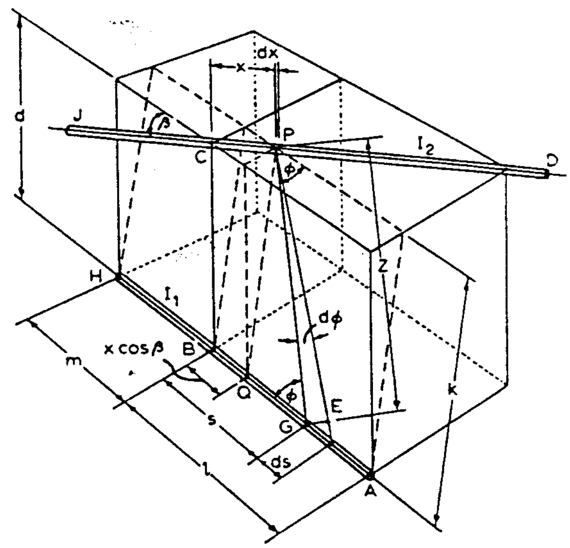


Figure E.6—Skewed conductors—β < 90˚, x and m positive

If this expression is integrated with respect to φ from $\phi = \phi_A$ (where G is at A) to $\phi = \phi_H$ (where G is at H), this will give the force dF on the element dx at P due to the current I_2 in this element and the current I_1 in the whole of conductor HA.

Thus

$$dF = \frac{I_1 I_2 dx 10^{-7} \sin\alpha}{k} \int_{\phi A}^{\phi H} \sin\phi d\phi$$

$$dF = \frac{I_1 I_2 dx 10^{-7} \sin\alpha}{k} (-\cos\phi) \begin{matrix} \phi H \\ \phi A \end{matrix}$$

i.e., the force F_p per meter at P is

$$\frac{dF}{dx} = \frac{I_1 I_2 10^{-7} \sin\alpha}{k} (\cos\phi_A - \cos\phi_H) \qquad \text{(E3)}$$

From the geometry of Figure E.6 it is easy to show that

$$\cos\phi_A = \frac{l - x\cos\beta}{\sqrt{[k^2 + (l - x\cos\beta)^2]}}$$

$$\cos\phi_H = \frac{-(m + x\cos\beta)}{\sqrt{[k^2 + (m + x\cos\beta)^2]}}$$

where $k = \sqrt{(d^2 + x^2 \sin^2\beta)}$

It can also be shown that

$$\sin\alpha = \sqrt{\frac{d^2\cos^2\beta + x^2\sin^2\beta}{d^2 + x^2\sin^2\beta}} \text{(see E.10.1)}$$

and substituting these values of cos ϕ_A, cos ϕ_H, k and sin α in Equation (E3), we obtain

$$F_p = \frac{I_1 I_2 10^{-7}\sqrt{(d^2\cos^2\beta + x^2\sin^2\beta)}}{d^2 + x^2\sin^2\beta} \times \left\{ \frac{l - x\cos\beta}{\sqrt{[d^2 + x^2\sin^2\beta + (l - x\cos\beta)^2]}} + \frac{m + x\cos\beta}{\sqrt{[d^2 + x^2\sin^2\beta + (m + x\cos\beta)^2]}} \right\} \quad \text{(E4)}$$

Figures E.7 and E.8 show that Equation (E4) applies also to cases in which β lies between 90˚ and 180˚ and when the dimensions m and x are negative.

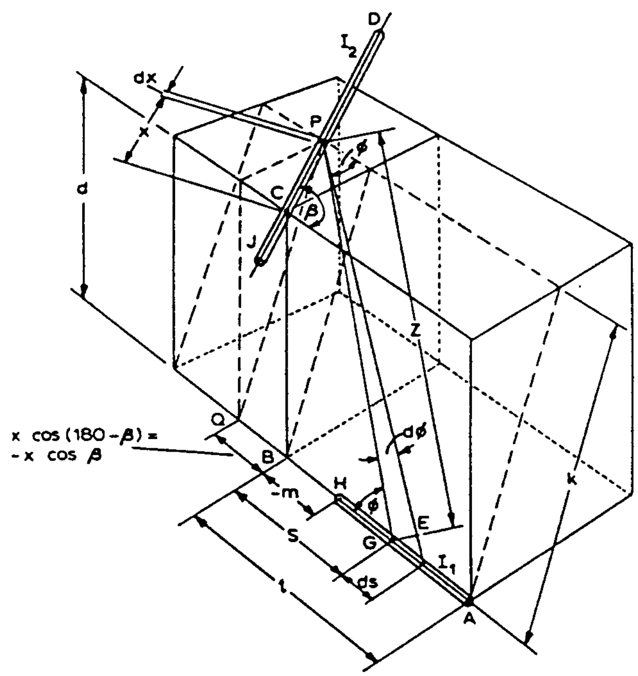


Figure E.7—Skewed conductors—β > 90˚, x positive, m negative

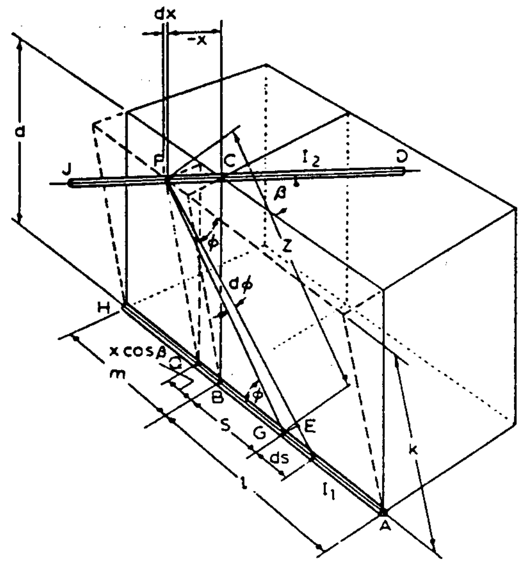


Figure E.8—Skewed conductors—β < 90˚, x negative, m positive

E.5.2 Direction of forces

It has already been stated in the discussion of Figure E.5 that the force per unit length at various points along the conductor JD acts at right angles to JD (i.e., radially) and that the force vector lies in the plane containing conductor HA and the point considered. The angle which the force vector makes with the plane in which conductor JD lies is given by

$$X = \tan^{-1}\left(\frac{d}{x\tan\beta}\right) \quad \text{(see E.10.2)} \tag{E5}$$

so that the orthogonal components of F_p are

$$F_h = F_p \cos X \tag{E6}$$

$$F_v = F_p \sin X \tag{E7}$$

E.6 Numerical example

To illustrate the use of Equations (E4) through (E7), consider the arrangement shown in Figure E.9. Two conductors are shown 30 cm and 80 cm long, crossing each other obliquely and forming part of a complete circuit carrying a current of 10^4 A. The shortest distance between the two conductors is along a line 10 cm long joining the middle point of the 80 cm conductor to a point 10 cm from one end of the 30 cm conductor.

The distribution of forces along the 80 cm conductor, computed from Equation (E4), is shown in Figure E.10 for six different angles of cross-over. It should be remembered that the forces acting on each differential length of conductor are uniplanar only in the cases for β = 0˚ (parallel) and β = 90˚ (right- angle cross-over) as shown by Equation (E5).

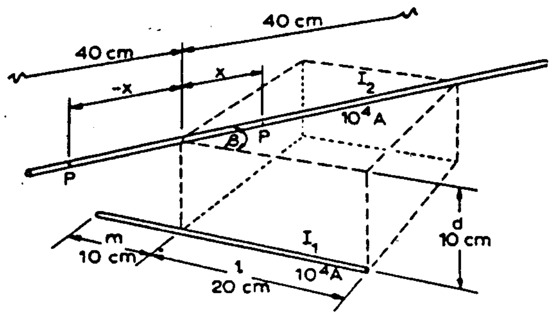


Figure E.9—Skewed conductors—numerical example

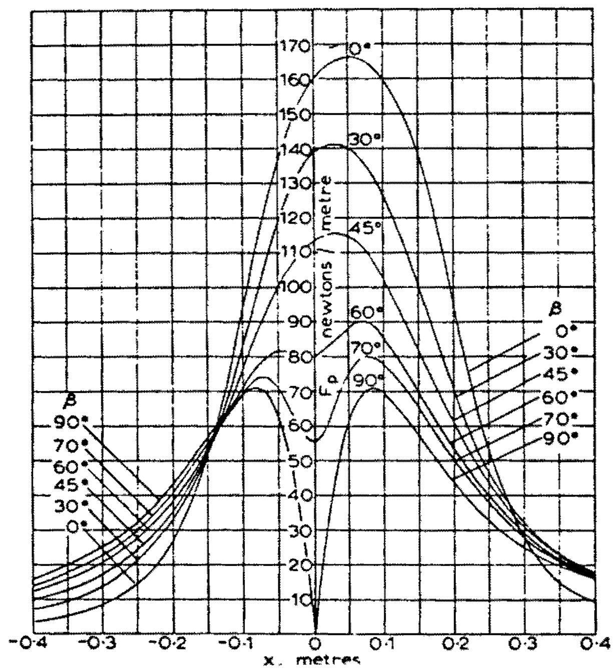


Figure E.10—Distribution of mechanical forces on skewed conductors for various angles of cross-over

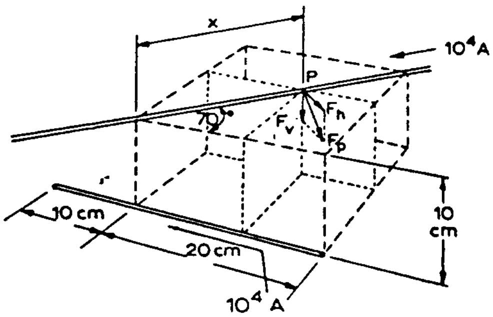


Figure E.11—70˚ cross-over

Note the transition in form of the curves from β = 0˚ to β = 90˚ in Figure E.10, the minimum points at $x = 0$ disappearing on curves where β < 45˚.

The component forces for the 70˚ cross-over (Figure E.11) are plotted in Figure E.12 from Equations (E6) and (E7), and it is these curves which would be used in calculating the total force and moments by graphical integration. The magnitude and direction of forces on supporting insulators may be deduced easily from the moments of the component forces by methods which are fully detailed in Frick, [1].

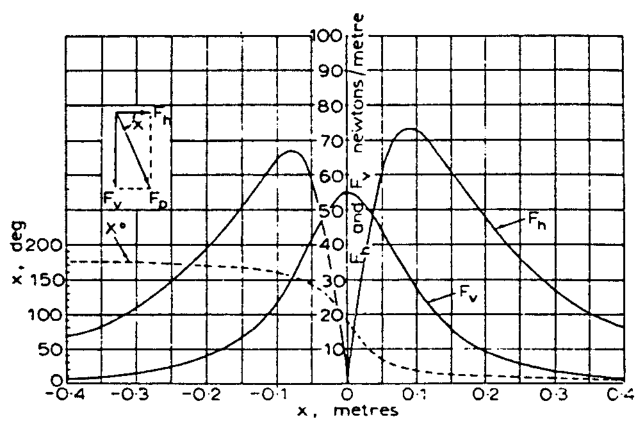


Figure E.12—Orthogonal components of mechanical forces on conductors with 70˚ cross-over

E.7 Special conductor arrangements

By suitable substitutions in Equation (E4), formulae may be obtained for the distribution of mechanical force in special conductor arrangements which agree with those already published.

E.7.1 Parallel conductors ($\beta = 0^\circ$)

a) Short conductors (see Figure E.13)

$$F_p = \frac{I_1 I_2 10^{-7}}{d}\left\{\frac{l-x}{\sqrt{[d^2+(l-x)^2]}} + \frac{m+x}{\sqrt{[d^2+(m+x)^2]}}\right\} \quad \text{(E8)}$$

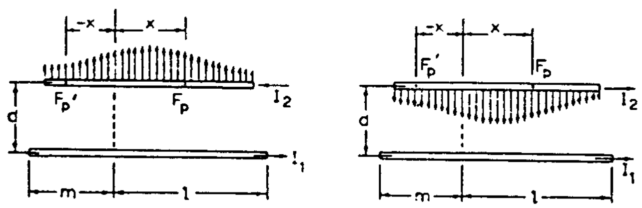


Figure E.13—Short parallel conductors

b) End of long conductor (see Figure E.14)

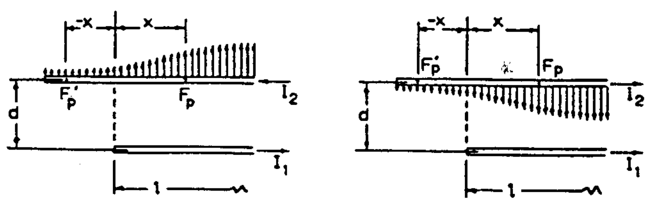


Figure E.14—End of long parallel conductor

If d and x are very small compared with l, and $m = 0$

$$\text{then} \quad \frac{l-x}{\sqrt{[d^2+(l-x)^2]}} = 1$$

$$\text{and} \quad \frac{m+x}{\sqrt{[d^2+(m+x)^2]}} = \frac{x}{\sqrt{(d^2+x^2)}}$$

Then from Equation (E4)

$$F_p = \frac{I_1 I_2 10^{-7}}{d}\left[1 + \frac{x}{\sqrt{(d^2+x^2)}}\right] \quad \text{(E9)}$$

c) Centre of long conductors

$m = l$, $x = 0$, and d is negligible compared with l. Then from Equation (E4),

$$F_p = \frac{2 I_1 I_2 10^{-7}}{d} \quad \text{(E9a)}$$

E.7.2 Right-angled cross-over (see Figure E.15) (β = 90°)

$$F_p = \frac{I_1 I_2 x 10^{-7}}{d^2 + x^2}\left[\frac{l}{\sqrt{(d^2 + x^2 + l^2)}} + \frac{m}{\sqrt{(d^2 + x^2 + m^2)}}\right] \quad \text{(E10)}$$

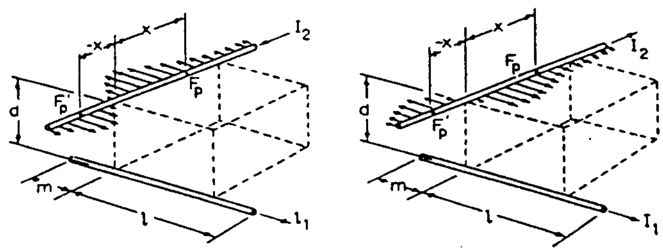


Figure E.15—90° cross-over

E.7.3 Right-angled bend (see Figure E.16) (β = 90° d = 0 m = 0)

From Equation (E10)

$$F_p = \frac{I_1 I_2 10^{-7}}{x}\left[\frac{l}{\sqrt{(x^2 + l^2)}}\right] \quad \text{(E11)}$$

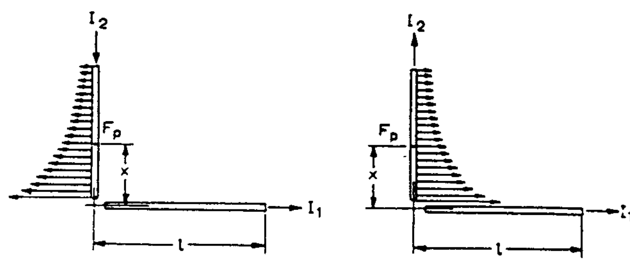


Figure E.16—90° bend

E.7.4 Any angle in a plane (see Figures E.17 and E.18) (d = 0)

$$F_p = \frac{I_1 I_2 10^{-7}}{x \sin\beta}\left[\frac{l - x\cos\beta}{\sqrt{(x^2 + l^2 - 2lx\cos\beta)}} + \frac{m + x\cos\beta}{\sqrt{(x^2 + m^2 + 2mx\cos\beta)}}\right] \quad \text{(E12)}$$

In Figure E.18, m is negative and β is in the second quadrant, so that cos β is also negative.

When m = 0 and β = 135°, then $\cos\beta = \frac{-\sqrt{2}}{2}$ and $\sin\beta = \frac{\sqrt{2}}{2}$

Let $l = \frac{x}{v}$

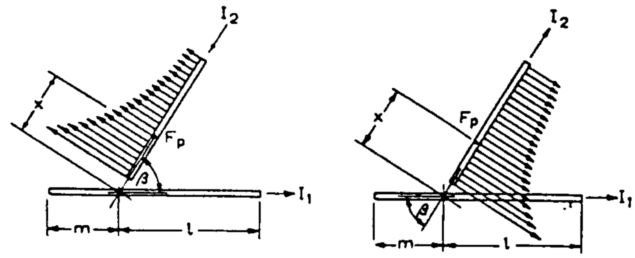


Figure E.17—Any angle in a plane < 90˚

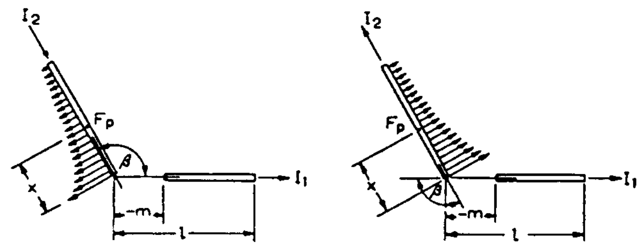


Figure E.18—Any angle in a plane > 90˚, m negative

Substituting in equation 12 we obtain

$$F_p = \frac{I_1 I_2 10^{-7}}{x}\left[\frac{\sqrt{2}+v}{\sqrt{(v^2+\sqrt{2}v+1)}}-1\right] \qquad \text{(E12a)}$$

which agrees with the formula given by Van Asperen [4].

Similarly when m = 0 and $\beta = 45°$, $\cos\beta = \sin\beta = \frac{\sqrt{2}}{2}$ and $l = \frac{x}{v}$, giving

$$F_p = \frac{I_1 I_2 10^{-7}}{x}\left[\frac{\sqrt{2}-v}{\sqrt{(v^2-\sqrt{2}v+1)}}+1\right] \qquad \text{(E12b)}$$

E.7.5 Forces at bends and corners of a conductor system

The standard Equations (E11) and (E12) for angled conductors lying in a plane do not take into account the non-uniform current distribution occurring near the junction of the conductors. As $x \rightarrow 0$ the current in the bend tapers off with a corresponding reduction in the mechanical forces in the vicinity of the corner. The problem is outside the scope of the paper, but an approximate solution may be obtained for a 90˚ bend by assuming that the force starts at the point $x = 0{\cdot}779r$, where r is the radius of the conductor. (Frick, [1])

E.8 Conclusions

So far as the author is aware, the general formulae developed in the paper have not previously been stated. They should prove useful to designers in circumstances where accuracy is important. In other cases, where approximate methods are appropriate, the rigid formulae may serve as a guide to the percentage error involved.

E.9 References

[1] Frick, C.W., "Electromagnetic forces on conductors with bends, short lengths and cross-overs," Gen. Elect. Rev., 1933, 36, p. 232.

[2] Chin, T. H., and Higgins, T. J., "Equations for evaluating short-circuit forces on multiple-strap single phase and polyphase busses for supplying low frequency induction furnaces," Trans Amer. Inst. Elect. Engrs, 1960, 79, Part II, p. 260.

[3] Higgins, T. J., "Formulas for calculating short circuit forces between conductors of structural shape," ibid., 1943, 62, p. 659.

[4] Van Asperen, C. H., "Mechanical forces on busbars under short circuit conditions," Ibid. 1922, 42, p. 1091.

[5] Dwight, H. B., "Repulsion between strap conductors," Elect. World, 1917, p. 522.

[6] Dunton, W. F., "Electromagnetic forces on current carrying conductors," J. Sci. Instrum., 1927, 4, p. 440.

[7] Dwight, H. B., "Calculation of magnetic force on disconnecting switches," Trans Amer. Inst. Elect. Engrs, 1920, 40, p. 1337.

E.10 Appendixes

E.10.1 To determine the angle between conductor JD and the direction of the magnetic flux (see Figure E.19)

Consider point P on conductor JD. The direction of the magnetic flux ϕ at this point due to the current I_1 in conductor HA is normal to the plane BPA and is indicated by the line PT. It is required to find the angle TPJ in terms of x, d and β.

Rectangular co-ordinate axes PX, PY, and PZ with P as the origin are reproduced in Figure E.19*a*, in which PC and PR represent the directions of the conductor JD and flux vector PT, respectively. From a well-known proposition in co-ordinate geometry the cosine of the angle between two lines is equal to the sum of the products of their respective direction-cosines.

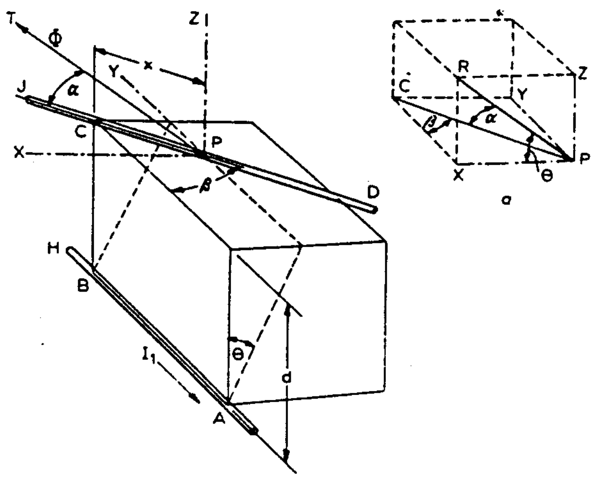


Figure E.19—Angle between conductor and direction of magnetic flux

Thus, if a, b and c are the direction-cosines of PC and a', b' and c' are the direction-cosines of PR, we have $\cos\alpha = aa' + bb' + cc'$

where

$a = \cos XPC = \sin\beta$ $\quad a' = \cos XPR = \cos\theta$
$b = \cos YPC = \cos\beta$ $\quad b' = \cos YPR = 0$
$c = \cos ZPC = 0$ $\quad c' = \cos ZPR = \sin\theta$

Then

$$\cos\alpha = \sin\beta\cos\theta \tag{E13}$$

Referring to Figure E.19 it is seen that

$$\cos\theta = \frac{d}{\sqrt{(d^2 + x^2\sin^2\beta)}}$$

Therefore

$$\cos\alpha = \frac{d\sin\beta}{\sqrt{(d^2 + x^2\sin^2\beta)}}$$

$$\sin\alpha = \sqrt{\left(1 - \frac{d^2\sin^2\beta}{d^2 + x^2\sin^2\beta}\right)}$$

$$\sin\alpha = \sqrt{\left(\frac{d^2\cos^2\beta + x^2\sin^2\beta}{d^2 + x^2\sin^2\beta}\right)} \tag{E14}$$

E.10.2 To determine the angle between the direction of the mechanical force on an element of conductor JD at point P and the normal to the conductor in the plane CPA´ (see Figure E.20).

The mechanical force on an element of conductor JD due to the currents I_1 in HA and I_2 in JD is at right angles to the direction of the magnetic flux at point P. It lies, therefore, in the plane BPA and is represented by the line PF. Produce PF to cut HA in T. Since the force PF is at right angles to conductor JD, its trace PS in the plane CPA´ is also normal to JD.

Then

$$\tan X = \frac{ST}{PS} = \frac{d}{x \tan \beta} \quad \text{(E15)}$$

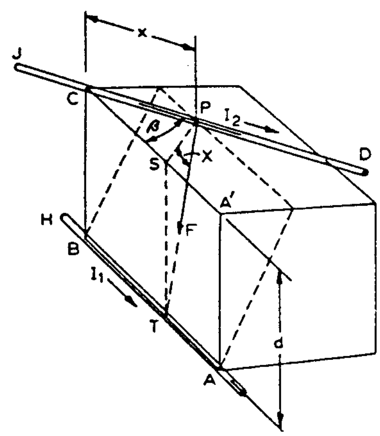


Figure E.20—Direction of mechanical force

Annex F

(informative)

Static analysis of substation rigid-bus structure

F.1 Introduction

Clause 11 of this guide provides good guidance for determining maximum span lengths of single-level bus conductors based on the allowable vertical deflection or conductors' fiber stress. However, it fails to address a two-level bus arrangement, the most common rigid-bus arrangement in a low-profile substation, where one bus at a lower-level bus supports the upper-level bus with an A-frame. In such an arrangement, the forces acting on the upper bus are transmitted to the lower bus as concentrated forces at each base of the A-frame. The lower bus may be subjected to severe stress due to these concentrated loads. This annex is intended to provide a simple statistical method for analyzing two-level bus configurations. Since the concern here is the higher fiber stress that can be developed at the base of the A-frame, only the fiber stress aspect is addressed.

F.2 Basis of static analysis

The bus conductors are subjected to uniformly distributed forces (weight, wind force, and fault current force) in vertical and horizontal directions. Some concentrated forces are transmitted through the A-frame from an upper bus as well. When external forces act upon the bus, a reaction (force) is developed at the insulator supports. These forces also create bending moments along the bus conductor. The statistical analysis of the bus structures determines the values of these unknown force reactions and moments.

The basis of the analysis for a bus conductor are the static equilibrium equations. Equilibrium equations relate the forces acting on the bus conductor with reactions and the bending moments developed in the bus conductor, and the deformation equations. The deformation equations are required to supplement the equilibrium equations to solve the statistically independent continuous bus structures. Most buses are supported by three or more supports. They normally form indeterminate continuous beams. The deformation equation that is applicable here for the continuous bus with simple end conditions is the *Three-Moment Theorem*. It provides the relations of moments at three supports of two adjacent spans of a continuous beam. (See Figure F.1.)

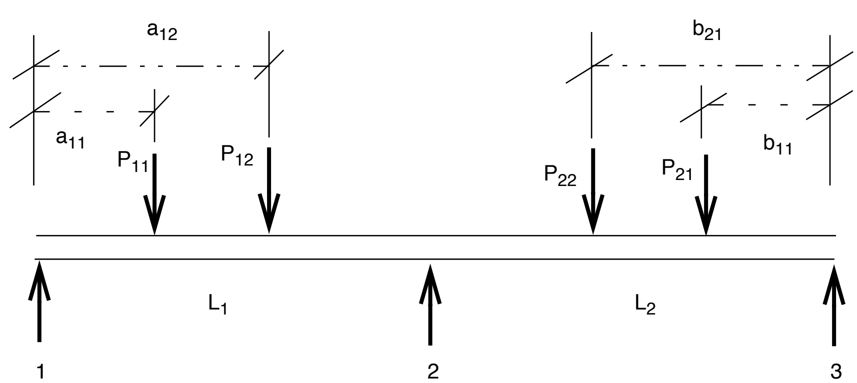


Figure F.1—Two adjacent spans of continuous beam

Using the Three-Moment Theorem and Figure F.1, the following equation can be derived:

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = -\left[\frac{w({L_1}^3 + {L_2}^3)}{4} + \frac{\sum(P_1a_1({L_1}^2 - {a_1}^2)}{L_1} + \frac{\sum(P_2a_2({L_2}^2 - {a_2}^2)}{L_2}\right] \quad \text{(F1)}$$

where

M_1, M_2, M_3 are moments at supports 1, 2, 3 in lbf/ft
L_1, L_2 are span lengths of two adjacent spans in ft
w is uniformly distributed loads in lbf/ft
a_1 is the distance of concentrated load(s) P_1 from insulator 1 in feet
b_2 is the distance of concentrated load(s) P_2 from insulator 3 in feet

To apply the Three-Moment Theorem, two moments at the end supports must be known. Normally the continuous bus is assumed to be pinned at the ends; thus, the end moments are zero. When the bus extends beyond the end support, the moment become non-zero due to cantilever bus section. In this case the end moment can be solved using the moment equation for the cantilever section. Three-Moment Theorem cannot be applied to bus configurations with fixed ends of unknown moments.

The step-by-step procedure for analyzing the bus conductor using the Three-Moment Theorem and the equilibrium equations is described in F.3. Samples of generalized equations are given in F.5 (for buses without concentrated loads) and F.6 (for buses with concentrated loads) for analyzing some common bus configurations.

In applying these procedures, the sign conventions for the forces and bending moments are shown in Table F.1.

Table F.1—Sign convention for the applied forces and bending moments

Force Direction	Sign
Vertical downward force in –Y-axis	Positive
Vertical uplift force in +Y-axis	Negative
Horizontal force in –X-axis	Positive
Horizontal force in + X-axis	Negative
Horizontal force in –Z-axis	Positive
Horizontal force in + Z-axis	Negative
Bending moments due to positive force	Negative
Bending moments due to negative force	Positive

Note that the bus with negative bending moments in the vertical direction has convex upward curvature at the point of a moment. Since the external weight, wind, and short-circuit forces are in X, Y, and Z directions, the analysis should be performed separately in each direction and combined vectorially to obtain resultant values.

F.3 Step-by-step method

The structural analysis of bus conductor can be performed in three stages. The first stage is to determine the maximum allowable span length of the bus based on either vertical deflection or fiber stress using the method outlined in this guide. The second and third stages are for analyzing more complex bus configurations involving double-level bus arrangements. The second stage analyzes the upper-level bus and calculates the values of the concentrated forces of the upper bus that are transmitted to the lower-level bus.

F.3.1 Stage 1 calculations for general bus structure

a) Lay out complete bus arrangement including main buses and feeder buses in each bay.
b) Determine bus-conductor sizes and their characteristics for each bus configuration.
c) Establish design parameters for each bus configuration (span lengths, spacing, A-frame height, A-frame base width, wind velocity, and short-circuit current).
d) Calculate conductor gravitational forces (weights of conductor, damping material, and ice according to Clause 8 of this guide. Sum the gravitation unit forces [Equation (13) of this guide] in vertical direction of each bus configuration.
e) Determine maximum allowable span length of the bus L_d for a given vertical deflection using one of the applicable equations given in 11.1 of this guide [Equations (14) through (21)] for each bus configuration.
f) Calculate conductor wind forces [Equation (9)] in horizontal (X-axis or Z-axis) per Clause 9 of this guide for each bus configuration.
g) Calculate conductor short-circuit forces [Equation (12)] in horizontal (X-axis or Z-axis) according to Clause 10 of this guide for each bus configuration.
h) Combine the vertical forces and the horizontal forces, and get the resultant force [Equation (22)] for each bus configuration.
i) Find the allowable span length of the bus L_S based on the fiber stress using one of the applicable equations given in 11.2 of this guide [Equations (23) through (29)] for each bus configuration.
j) Check that the longest span utilized in the bus configuration is less than the calculated maximum allowable span length of the bus (L_A). If not, change the bus configuration and repeat the above steps for the new configuration.
k) For single-level bus arrangement without concentrated loads, the analysis ends here. For two-level bus using A-frame supports proceed to the second stage.

F.3.2 Stage 2 calculations for upper-level bus structure

The following steps are required to determine the force for the upper-level bus:

a) Find the per unit total vertical and horizontal (X-axis) direction. See Figure F.2.
b) Calculate the moment at each support (A-frame or insulator support) by writing the equation for Three-Moment theorem [Equation (F1)] consecutively for each two adjacent spans and by solving the simultaneous equations. Moments should be calculated separately for vertical and horizontal (X-axis) direction. See Figure F.2

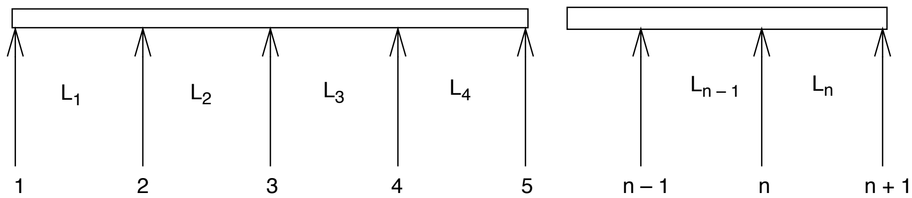


Figure F.2—Stage 2 calculations

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = \frac{-w(L_1^3 + L_2^3)}{4}$$

$$M_2L_2 + 2M_3(L_2 + L_3) + M_4L_3 = \frac{-w(L_2^3 + L_3^3)}{4}$$

....

....

$$M_{n-2}L_{n-2} + 2M_{n-1}(L_{n-2} + L_{n-1}) + M_nL_{n-1} = \frac{-w(L^3_{n-2} + L^3_{n-1})}{4}$$

$$M_{n-1}L_{n-1} + 2M_n(L_{n-1} + L_n) + M_{n+1}L_n = \frac{-w(L_{n-1}^3 + L_n^3)}{4}$$

There will be n–1 equations for bus with n sections and n+1 supports. Normally end moments M_1 and M_{n+1} are zero for pinned supports or can be easily figured out for continuous cantilever supports.

c) Determine vertical and horizontal reactions at each support by solving moment equilibrium Equation (F2):

Moment at a point = moments due to distributed loads plus moments due to concentrated loads.

$$= \frac{wL^2}{2} + \sum Px \qquad \text{(F2)}$$

where

$\frac{wL^2}{2}$ = moment due to distributed load

Px = moments due to concentrated load

where

w = Generalized distributed load in one direction in lbf per ft
P = Generalized concentrated load(s) in same direction in lbf
L = Span length of the bus conductor on one side of moment point in ft
x = Distance of concentrated load(s) from moment point in ft

For example write moment at second support to solve the reaction R_1 as follows:

$$M_2 = -\left(\frac{wL_1^{\ 2}}{2} + R_1 L_1\right) \qquad R_1 = \frac{(M_2 + wL_1^{\ 2}/2)}{L_1}$$

d) Calculate moments at other points preferably midpoints of the bus using the same moment equilibrium to determine the location of maximum and minimum moments along the bus conductor. These calculations may be needed to locate the welds at the minimum stress points.

$$Rm = \sqrt{Vm^2 + Hm^2} \tag{F3}$$

where

Rm = Resultant moment
Vm = Vertical moment
Hm = Horizontal moment

e) Combine vertical and horizontal moments and get bending moments for upper bus [Equation (F3)].

f) Find maximum resultant moment and determine the maximum fiber stress [Equation (F4)] at that point. The maximum stress normally occurs at the second last support.

$$\text{Maximum fiber stress} = \text{Max. moment} \times 12/S \tag{F4}$$

where
S is the section modulus in³.

g) Check that the calculated fiber stress is less than the maximum allowable stress (minimum yield strength of bus material).

h) Determine the vertical (Y-axis) and horizontal (X-axis) concentrated forces that will be transmitted to low bus at a particular A-frame support (See Figure F.3).

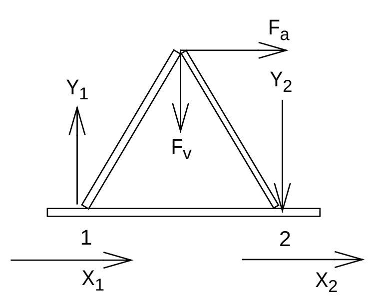


Figure F.3—Upper-level bus

$$Y_1 = \frac{F_v}{2} + \left(\frac{F_h}{2}\right) + \left(\frac{H_A}{L_A}\right) \tag{F5}$$

$$Y_2 = \frac{F_v}{2} + \left(\frac{F_h}{2}\right) + \left(\frac{H_A}{L_A}\right) \tag{F6}$$

where

Y_1 = concentrated vertical force at base one of A-frame
Y_2 = concentrated vertical force at base two of A-frame

$$X_1 = \frac{F_h}{2} + \left(\frac{F_v}{2}\right) + \left(\frac{L_A}{H_A}\right) \tag{F7}$$

$$X_2 = \frac{F_h}{2} + \left(\frac{F_v}{2}\right) + \left(\frac{L_A}{H_A}\right) \tag{F8}$$

X_1 = concentrated horizontal force at base one of A-frame
X_2 = concentrated horizontal force at base two of A-frame

where

F_v = vertical force
= – vertical reaction at the top of A-frame
F_h = horizontal force
= – horizontal reaction at the top of A-frame
H_A = height of A-frame in ft
L_A = half of base of A-frame in ft

i) Calculation for high bus ends here. Proceed to F.3.3 for lower bus structure.

F.3.3 Stage 3 calculations for lower-level bus structure

The following steps are required to determine the forces for the lower-level bus:

a) Start with unit total vertical and horizontal forces for lower-level bus. The horizontal force should be in the Z-axis direction.
b) Consider the vertical forces transmitted from high bus to each base of the A-frame [Equations (F5) and (F6)] as concentrated load.
c) Calculate the moment at each insulator support by writing equations for Three-Moment Theorem [Equation (F1)] consecutively for each two adjacent spans and by solving the simultaneous equations. Moments should be calculated separately for vertical and horizontal (Z-axis) directions. Moment calculation for the horizontal direction is identical to step b) of F.3.2. However, the moment equation in the vertical direction for a span involving an A-frame would include terms for concentrated vertical loads. For example moment equations for bus structure with A-frame resting on the second and third spans (A-frame over insulator no. 3 see Figure F.4) will be as follows:

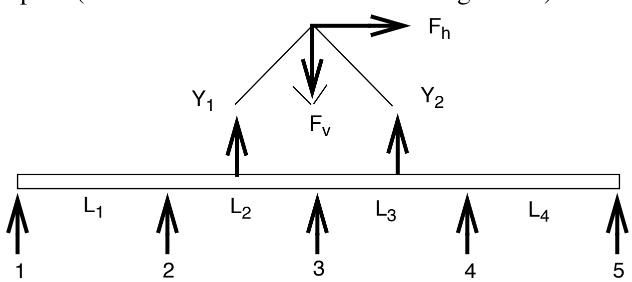


Figure F.4—Lower-level bus

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = -\left[\frac{w(L_1^{\ 3} + L_2^{\ 3})}{4} + \frac{Y_1L_A(L_2^{\ 2} - L_A^{\ 2})}{L_2}\right]$$

$$M_2L_2 + 2M_3(L_2 + L_3) + M_4L_3 = -\left[\frac{w(L_2^{\ 3} + L_3^{\ 3})}{4} + \frac{Y_1(L_2 - L_A)(L_2^{\ 2} - L_A^{\ 2})}{L_2}\right.$$

$$\left. + \frac{Y_2(L_3 - L_A)(L_3^{\ 2} - (L_3 - L_A)^2}{L_3}\right]$$

$$M_3L_3 + 2M_4(L_3 + L_4) + M_5L_4 = -\left[\frac{w(L_3^{\ 3} + L_4^{\ 3})}{4} + \frac{Y_2L_A(L_3^{\ 2} - L_A^{\ 2})}{L_3}\right]$$

....

$$M_{n-1}L_{n-1} + 2M_n(L_{n-1} + L_n) + M_{n+1}L_n = -\frac{w(L_{n-2}^{\ 3} + L_{n-1}^{\ 3})}{4}$$

where Y_1 and Y_2 are vertical forces transmitted at the base of A-frame and L_A is the distance of the base number one and two from support number three (3) (half of A-frame base).

Normally end moments M_1 and M_n are zero for pinned supports or can be easily calculated for continuous cantilever supports.

d) Calculate vertical and horizontal reactions at each insulator support using moment equilibrium equations, similar to step c) of F.3.2.
e) Calculate moments at other points preferably midpoints of the bus spans and base of each A-frame using the same moment equilibrium equations to determine the point of maximum and minimum moments.
f) Combine vertical and horizontal moments and get resultant bending moments for lower bus [Equation (F3)].
g) Find maximum resultant moment among all moments and determine the maximum fiber stress [Equation (F4)]. The maximum moment will usually be developed at one of the bases of the A-frame.
h) Check that the calculated maximum fiber stress is less than the maximum allowable stress (minimum yield strength of the bus material). The maximum fiber stress will occur at the base of A-frame where the frame is welded. Determination must be made whether to consider the effects of welding and to reduce the maximum yield strength at the welding point.
i) This is the end of bus strength calculations. Proceed to calculate the insulator cantilever requirements as outlined in F.4.

F.4 Insulator cantilever forces

Clause 12 of this guide provides simplified equations for calculating insulator cantilever forces as a function of effective bus span length as given in Table 5. The effective span length as defined in Table 5 of this guide is applicable only to equal span bus length and concentrated loads. The bus analysis should be made according to the procedures described in F.3 above to determine the horizontal reactions.

The horizontal reactions calculated are the bus forces on the insulators. The span lengths are already accounted for in the reaction calculations made in step B3 or C4. These bus forces do not account for the overload factors given in the guide. Since the calculation is tedious, this method of adjusting the bus reactions is preferred. The adjustment factor is

$$K_a = \frac{(K_1 F_w K_2 F_{SC})}{(F_{SC} + F_W)} \quad \text{(F9)}$$

where

K_1 = overload factor for wind force

K_2 = overload factor for short circuit force

F_W = unit wind force

F_{SC} = unit short circuit force

The complete equation [Equation (35)] for the total cantilever load on a vertically mounted insulator supporting a horizontal bus becomes

$$F_{IS} = \frac{K_1 F_{WI}}{2} + \frac{K_a R_i (H_i H_F)}{H_1} \quad \text{(F10)}$$

where

F_{WI} = wind force on insulator

R_i = adjusted horizontal bus reaction for a support point (see Figure F.11)

H_i = insulator height in inches

H_f = bus center height above insulator in inches

K_a = adjustment factor for overload

When calculating the insulator cantilever strength for low bus in double bus arrangement, the concentrated horizontal force transmitted from the upper bus (in X-axis) should be included in the horizontal bus reaction R_i. The resultant R_i can be calculated using Equation (F11). The X-axis force is assumed to be divided among the low-bus insulators.

$$R_i = \sqrt{\left[\frac{(X_1 + X_2)}{N}\right]^2 + R^2} \quad \text{(F11)}$$

where

X_1 and X_2 = concentrated horizontal forces [Equations (F7) and (F8)] from upper bus

N = number of insulators in low bus

R = calculated horizontal bus reaction for a support point

R_i = adjusted horizontal bus reaction for a support point

F.5 Examples for common bus configurations without concentrated load

F.5.1 Single-span bus, two pinned supports

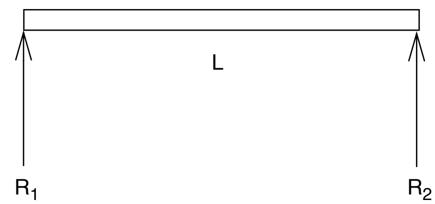


Figure F.5—Single-span bus, two pinned supports

$M_1 = 0$

$M_2 = 0$

$$R_1 = -\frac{(wL)}{2} \quad R_1 = -\frac{(wL)}{2} \quad M_{\max} = -\frac{(wL^2)}{8S} \quad FS_{\max} = -\frac{(wL^2)}{8S}$$

where

w = generalized distributed unit force in lbf per ft
L = span length in feet
M_1 and M_2 = moments at supports 1 and 2 in lbf
R_1 and R_2 = reactions at supports 1 and 2 in lbf
S = section modulus of bus conductor
FS = fiber stress in lbf per in^3

F.5.2 Single-span bus with cantilever on one side, one pinned and one continuous support

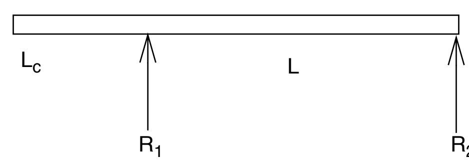


Figure F.6—Single-span bus, one pinned and one continuous support

$$M_1 = -\frac{(wL_c{}^2)}{2} \qquad M_2 = 0$$

$$R_1 = -\left(\frac{w(L + L_c)^2}{2L}\right) \quad R_2 = -\left(\frac{w(L^2 + L_c{}^2)}{2L}\right) \quad M_{mid} = \frac{wL^2}{8} - \frac{wL_c{}^2}{4}$$

$$M_{max} = \frac{wL^2}{8} - \frac{wL_c^2}{4} \quad \text{at midpoint of the bus span}$$

$$M_{max} = -\frac{wL_c^2}{2} \quad \text{at support 1}$$

$$FS_{max} = \frac{(12M_{max})}{S}$$

where

w = generalized distributed unit force in lbf per ft
L = span length in feet
L_c = protruding length of cantilever
M_1 and M_2 = moments at supports 1 and 2 in lbf
R_1 and R_2 = reactions at supports 1 and 2 in lbf
S = section modulus of bus conductor
FS = fiber stress in lbf per in^2

F.5.3 Single-span bus, two fixed supports

Figure F.7—Single-span bus, two fixed supports

$$M_1 = -\left(\frac{wL^2}{12}\right) \quad M_2 = -\left(\frac{wL^2}{12}\right) \quad M_{mid} = \frac{wL^2}{24}$$

$$R_1 = -\left(\frac{wL}{2}\right) \quad R_2 = -\left(\frac{wL}{2}\right) \quad M_{max} = \frac{wL^2}{12} \text{ at end support of the bus support}$$

$$FS_{max} = \frac{wL^2}{S}$$

where

w = generalized distributed unit force in lbf per ft
L = span length in feet
M_1 and M_2 = moments at supports 1 and 2 in lbf
R_1 and R_2 = reactions at supports 1 and 2 in lbf
S = section modulus of bus conductor
FS = fiber stress in lbf per in^2

F.5.4 Continuous two-span bus, two pinned and one continuous support

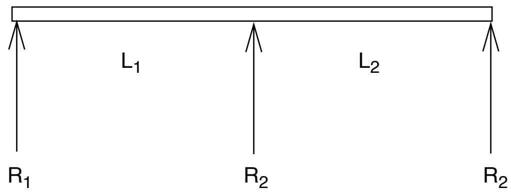


Figure F.8—Continuous two-span, two-pinned and one continuous support

$$M_1 = 0 \qquad M_2 = -\frac{w(L_1{}^3 + L_2{}^3)2(L_1 + L_2)}{4} \qquad M_3 = 0$$

$$R_1 = -\frac{M_2 + \dfrac{wL_1{}^2}{2}}{L_1} \qquad R_2 = -\left[\frac{w(L_1 + L_2)^2}{2} + \frac{R_1(L_1 + L_2)}{L_2}\right] \qquad R_3 = -\frac{M_2 + \dfrac{wL_2{}^2}{2}}{L_2}$$

$$M_{\mathrm{mid1}} = -\left[\frac{w\left(\dfrac{L_1}{2}\right)^2}{2} + R_1\left(\frac{L_1}{2}\right)\right] \qquad M_{\mathrm{mid2}} = -\left[\frac{w\left(\dfrac{L_2}{2}\right)^2}{2} + R_3\left(\frac{L_2}{2}\right)\right]$$

$$FS_{\max} = \frac{(12M_{\max})}{S} \qquad M_{\max} = -\frac{w(L_1{}^3 + L_2{}^3)2(L_1 + L_2)}{4} \text{ at mid-support}$$

where

w = generalized distributed unit force in lbf per ft

L_1 and L_2 = span lengths in feet

M_1, M_2 and M_3 = moments at supports 1, 2, and 3 in lbf

R_1, R_2 and R_3 = reactions at supports 1, 2, and 3 in lbf

S = section modulus of bus conductor

FS = fiber stress in lbf per in^2

For $L_1 = L_2$:

$$M_1 = 0 \qquad M_2 = -\left(\frac{wL^2}{8}\right) \qquad M_3 = 0$$

$$R_1 = -\left(\frac{3}{8}\right)wL \qquad R_2 = -\left(\frac{5}{4}\right)wL \qquad R_3 = -\left(\frac{3}{8}\right)wL$$

F.5.5 Continuous three-span bus, two pinned and two continuous supports

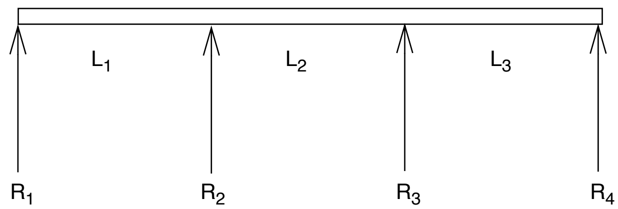


Figure F.9—Continuous three-span bus, two pinned and two continuous supports

$$M_1 = 0 \qquad M_4 = 0$$

$$M_2 = [2(L_1 + L_2)] + M_3 L_2 = -\frac{w({L_1}^3 + {L_2}^3)}{4} \qquad M_2 L_2 + M_3[2(L_2 + L_3)] = -\frac{w({L_2}^3 + {L_3}^3)}{4}$$

$$R_1 = -\left(M_2 + \frac{w{L_1}^2}{2}\right)L_1 \qquad R_2 = -\frac{\left[M_3 + \frac{w(L_1 + L_2)^2}{2} + R_1(L_1 + L_2)\right]}{L_2}$$

$$R_3 = -\frac{\left[M_4 + \frac{w(L_1 + L_2 + L_3)^2}{2} + R_1(L_1 + L_2 + L_3) + R_2(L_2 + L_3)\right]}{L_3}$$

$$R_4 = -\left(M_3 + \frac{w{L_3}^2}{2}\right)L_3$$

$$M_{\text{mid1}} = -\left[\frac{w\left(\frac{L_1}{2}\right)^2}{2} + \frac{R_1 L_1}{2}\right] \qquad M_{\text{mid2}} = -\left[\frac{w\left(L_1 + \frac{L_2}{2}\right)^2}{2} + R_1\left(L_1 + \frac{L_2}{2}\right) + \frac{R_2 L_2}{2}\right]$$

$$M_{\text{mid3}} = -\left[\frac{w\left(\frac{L_2}{2}\right)^2}{2} + \frac{R_3 L_2}{2}\right] \qquad M_{\text{max}} = M_2 \; or \; M_2 \text{ at second or third support}$$

where

w = generalized distributed unit force in lbf per ft
L_1, $L_{2,}$ and L_3 = span lengths in feet
M_1, M_2, $M_{3,}$ and M_4 = moments at supports 1, 2, 3, and 4 in lbf
R_1, R_2, $R_{3,}$ and R_4 = reactions at supports 1, 2, 3, and 4 in lbf
S = section modulus of bus conductor
FS = fiber stress in lbf per in^2

For $L_1 = L_2 = L_3 = L$ (for three equal spans)

$$M_1 = 0 \quad M_2 = -\left(\frac{wL^2}{10}\right) \quad M_3 = -\left(\frac{wL^2}{10}\right) \quad M_4 = 0$$

$$R_1 = -\left(\frac{4}{10}\right)wL \quad R_2 = -\left(\frac{11}{10}\right)wL \quad R_3 = -\left(\frac{11}{10}\right)wL \quad R_4 = -\left(\frac{4}{10}\right)wL$$

$$\mathrm{M}_{\max} = M_2 \text{ or } M_3 = -wL^2/10$$

$$FS_{\max} = \frac{-12(wL^2)}{10S}$$

F.5.6 Continuous four equal spans, two pinned and three fixed supports

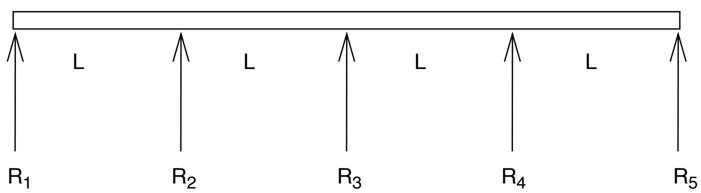


Figure F.10—Continuous four spans, two pinned and three fixed supports

$$M_1 = 0 \quad M_5 = 0$$

$$M_2 = -\left[\frac{\frac{w(L^3 + L^3)}{4} + M_3 L}{2(L + L)}\right] \quad = -\left(\frac{3}{28}\right)wL^2 \quad M_3 = -\left(\frac{2}{28}\right)wL^2 \quad M_4 = -\left(\frac{2}{28}\right)wL^2$$

$$R_1 = -\left(\frac{11}{28}\right)wL^2 \quad R_2 = -\left(\frac{32}{28}\right)wL^2 \quad R_3 = -\left(\frac{26}{28}\right)wL^2 \quad R_4 = -\left(\frac{32}{28}\right)wL^2$$

$$R_5 = -\left(\frac{32}{28}\right)wL^2 \quad M_{\max} = M_2 \; or \; M_4 = \left(\frac{3}{28}\right)wL^2 \text{ at second or fourth support} \quad FS_{\max} = \left(\frac{36}{28S}\right)wL^2$$

F.6 Example for common bus configuration with concentrated load

F.6.1 Single-span bus with A-frame on cantilever side, one pinned and one continuous support

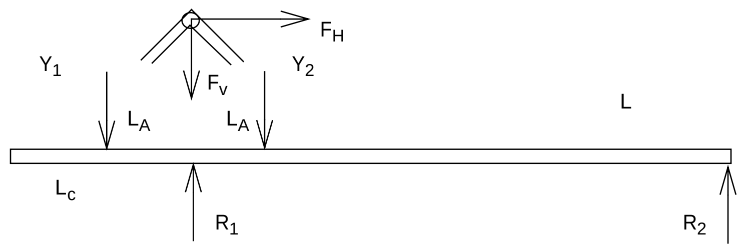


Figure F.11—Single-span bus with A-frame one pinned and one continuous support

$$M_1 = -\left[\frac{wL_C^{\ 2}}{2} + Y_1 L_A\right] \qquad R_1 = -\frac{\left[\frac{wL_1 + L_C^{\ 2}}{2} + Y_1(L_A + L) + Y_2(L - L_A)\right]}{L}$$

$$M_2 = 0 \qquad R_2 = -\frac{\left[M_1 + \frac{wL^2}{2} + Y_1 L_A\right]}{L}$$

$$M_{\text{mid}} = -\left[\frac{wL^2}{8} + \frac{R_2 L}{2}\right] \qquad M_{Y1} = -\frac{w(L_C + L_A)^2}{2} \qquad M_{Y2} = -\left[\frac{w(L_C + L_A)^2}{2} + R_1 L_A + 2Y_1 L_A\right]$$

$$M_{\max} = M_{y2} \qquad FS_{\max} = \frac{12M_{\max}}{S}$$

where

w = generalized distributed unit force in lbf per ft
L = span lengths in feet
L_A = length of half base of A-frame in ft
L_C = protruding length of cantilever
M_1, M_2 = moments at supports 1, 2 in lbf ft
R_1, R_2 = reactions at supports 1 2, in lbf ft
S = section modulus of bus conductor
FS = fiber stress in lbf per in^2

F.6.2 Two spans with A-frame on cantilever side, one pinned and two fixed supports

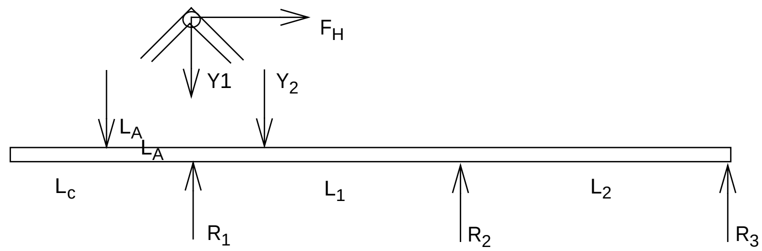


Figure F.12—Two spans with A-frame, one pinned and two fixed supports

$$M_1 L_1 + M_2[2(L_1 + L_2)] = -\left[\frac{wL_1 + L_2}{4} + \frac{Y_2 L_A (L_1{}^2 - L_A{}^2)}{L_1}\right] \quad M_1 = -\left[\frac{wL_C{}^2}{2} + Y_1 L_A\right] \quad M_3 = 0$$

$$R_1 = -\left[\frac{M_2 + \frac{w(L_1 + L_C)^2}{2} + Y_1(L_A + L_1) + Y_2(L_1 - L_A)}{L_1}\right]$$

$$R_2 = -\frac{\left[\frac{w(L_1 + L_2 + L_C)^2}{2} + R_1(L_1 + L_2) + Y_1(L_A + L_1 + L_2) + Y_2(L_1 + L_2 - L_A)\right]}{L_2}$$

$$R_3 = -\left(M_2 + \frac{wL_2{}^2}{2}\right)L_2$$

$$M_{\text{mid1}} = -\left[\frac{w\left(\frac{L_1}{2} + L_C\right)^2}{2} + R_1\left(\frac{L_1}{2}\right) + Y_1\left(L_A + \frac{L_1}{2}\right) + Y_2\left(\frac{L_1}{2} - L_A\right)\right] \quad M_{\text{mid2}} = -\left[\frac{w\left(\frac{L_2}{2}\right)^2}{2} + R_3\frac{L_2}{2}\right]$$

$$M_{Y1} = -\left[\frac{w(L_C - L_A)^2}{2}\right] \quad M_{Y_2} = -\left[\frac{w(L_C + L_A)^2}{2} + R_1 L_A + Y_1(2L_A)\right] \quad M_{\max} = M_{Y_2}$$

$$FS_{\max} = \frac{12M_{\max}}{S}$$

where

w = generalized distributed unit force in lbf per ft
L_1 and L_2 = span lengths of two spans in feet
L_A = length of half base of A-frame in ft
L_C = protruding length of cantilever
M_1, M_2, and M_3 = moments at supports 1, 2, and 3 in lbf ft
R_1, R_2 and R_3 = reactions at supports 1, 2, and 3 in lbf ft
S = section modulus of bus conductor
FS = fiber stress in lbf per in^2

F.6.3 Two-span bus with A-frame on mid-support—two pinned and one continuous support

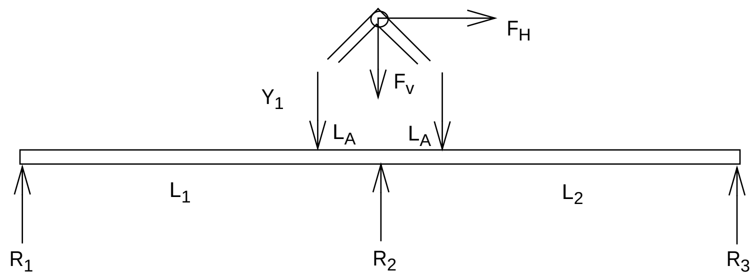


Figure F.13—Two span bus with A-frame, two pinned and one continuous support

$$M_1 = 0 \quad M_3 = 0 \quad M_2(2(L_1+L_2)) = -\left[\frac{w(L_1^{\ 3}+L_2^{\ 3})}{4} + Y_1(L_1-L_A)\left(L_1^{\ 2} - \frac{(L_1-L_A)^2}{L_1}\right) + Y_2(L_2-L_A)\left(L_2^{\ 2} - \frac{(L_2-L_A)^2}{L_2}\right)\right]$$

$$R_1 = -\frac{\left[M_2 + w\left(\frac{L_1^{\ 2}}{2}\right) + Y_1 L_A\right]}{L_1} \qquad R_2 = -\frac{\left[\frac{w(L_1+L_2)^2}{2} + R_1(L_1+L_2) + Y_1(L_A+L_2) + Y_2(L_2-L_A)\right]}{L_2}$$

$$R_3 = -\frac{\left[M_2 + w\left(\frac{L_2^{\ 2}}{2}\right) + Y_2 L_A\right]}{L_2} \qquad M_{\text{mid1}} = -\left[\frac{w\left(\frac{L_1}{2}\right)^2}{2} + R_1\left(\frac{L_1}{2}\right)\right] \qquad M_{\text{mid2}} = -\left[\frac{w\left(\frac{L_2}{2}\right)^2}{2} + R_3\left(\frac{L_2}{2}\right)\right]$$

$$M_{Y1} = -\left[\frac{w(L_1-L_A)^2}{2} + R_1(L_1+L_A)\right] \qquad M_{Y2} = -\left[\frac{w(L_2-L_A)^2}{2} + R_3(L_2-L_A)\right]$$

$M_{\max}$ = max. among all moments = M_{Y1} or M_{Y2}

$$FS_{\max} = \frac{(12M_{\max})}{S}$$

where

w = generalized distributed unit force in lbf per ft
L_1 and L_2 = span lengths of two spans in feet
L_A = length of half base of A-frame in ft
L_C = protruding length of cantilever
Y_1 and Y_2 = concentrated forces transmitted from upper bus
M_1, M_2, and M_3 = moments at supports 1, 2, and 3 in lbf ft
R_1, R_2, and R_3 = reactions at supports 1, 2, and 3 in lbf ft
S = section modulus of bus conductor
FS = fiber stress in lbf per in^2